Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV EXAMINATION – SUMMER 2020

Subject Code: 3140610 Date:02/11/2020

Subject Name: Complex Variables and Partial Differential Equations

Time: 10:30 AM TO 01:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Show that the function $u = x^2 - y^2$ is harmonic and find the corresponding analytic function.	**03**
	(b)	Find the fourth roots of -1.	**04**
	(c)	(i) Find the image of the infinite strip $0 \le x \le 1$ under the transformation $w = iz + 1$. Sketch the region.	**03**
		(ii) Write the function $f(z) = z + \frac{1}{z}$ in the form $f(z) = u(r,\theta) + iv(r,\theta)$.	**04**
Q.2	**(a)**	Evaluate $\int_C (x^2 + ixy)dz$ from (1,1) to (2,4) along the curve $x = t, y = t^2$.	**03**
	(b)	Find the bilinear transformation which transforms $z = 2,1,0$ into $w = 1,0,i$	**04**
	(c)	(i) Evaluate $\oint_C \frac{dz}{z^2 + 1}$, where C is $\lvert z + i \rvert = 1$, counter clockwise.	**03**
		(ii) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} z^n$.	**04**
		OR	
	(c)	(i) Find the roots of the equation $z^2 + 2iz + (2 - 4i) = 0$.	**03**
		(ii) Find the roots of $\log z = i\frac{\pi}{2}$.	**04**
Q.3	**(a)**	Find $\oint_C \left(\frac{3}{z - i} - \frac{6}{(z - i)^2} \right) dz$, where $C : \lvert z \rvert = 2$.	**03**
	(b)	Find the residues of $f(z) = \frac{1}{(z-1)^2 (z-3)}$, has a pole at z = 3 and a pole of order 2 at z = 1.	**04**
	(c)	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series for the regions $(i) \lvert z \rvert < 1$, $(ii) 1 < \lvert z \rvert < 3$, $(iii) \lvert z \rvert > 3$.	**07**
		OR	
Q.3	**(a)**	Evaluate $\oint_C \frac{e^z}{z + i} dz$, where $C : \lvert z - 1 \rvert = 1$.	**03**
	(b)	Evaluate by using Cauchy's residue theorem $\int_C \frac{5z - 2}{z(z-1)} dz$; $\lvert z \rvert = 2$.	**04**

(c) Expand $f(z) = \dfrac{1}{(z+1)(z-2)}$ in Laurent's series for the regions $(i)|z| < 1$, $(ii) 1 < |z| < 2$, $(iii)|z| > 2$. **07**

Q.4 **(a)** Solve $yq - xp = z$. **03**

(b) Derive partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$. **04**

(c) (i) Solve the p.d.e. $r - 3as + 2a^2 t = 0$. **03**

(ii) Find the complete integral of $p(1+q) = qz$. **04**

OR

Q.4 **(a)** Find the solution of $(y-z)p + (z-x)q = x - y$. **03**

(b) Form the partial differential equation by eliminating the arbitrary function from $\phi(x+y+z, x^2+y^2+z^2) = 0$. **04**

(c) (i) Solve the p.d.e. $s + p - q = z + xy$. **03**

(ii) Solve by Charpit's method $q = 3p^2$. **04**

Q.5 **(a)** Solve $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$ **03**

(b) Solve the p.d.e. $u_x = 4u_y, u(0,y) = 8e^{-3y}$ using the method of separation of variables. **04**

(c) Find the solution of the wave equation $u_{tt} = c^2 u_{xx}$, $0 \le x \le L$ with the conditions $u(0,t) = u(L,t) = 0; t > 0$, $u(x,0) = \dfrac{\pi x}{L}, u_t(x,0) = 0; 0 \le x \le L$. **07**

OR

Q.5 **(a)** Solve the p.d.e. $r + s + q - z = 0$. **03**

(b) Solve $xu_x - 2yu_y = 0$ using the method of separation of variables. **04**

(c) Find the solution of $u_t = c^2 u_{xx}$ together with the initial and boundary conditions $u(0,t) = u(l,t) = 0; t \ge 0$ and $u(x,0) = \sin\dfrac{\pi x}{l}; 0 \le x \le l$. **07**