GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2021

Subject Code:3140610   **Date:07/09/2021**

Subject Name:Complex Variables and Partial Differential Equations

Time:02:30 PM TO 05:00 PM   **Total Marks: 70**

Instructions:
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

			Marks
Q.1	**(a)**	Find the real and imaginary parts of $f(z) = \mathrm{i}z + 2\bar{z}$.	03
	(b)	State De-Movire's formula and evaluate $(1 + i)^{96} + (1 - i)^{96}$.	04
	(c)	Define analytic function. Show that $u(x, y) = x^2 - y^2 - y$ is harmonic function, find a harmonic conjugate $v(x, y)$.	07
Q.2	**(a)**	Find the image of the region $|z| < 2$ under the transformation $w = 3z + i$.	03
	(b)	Find all solution of $sinz = 2$.	04
	(c)	Expand $f(z) = \frac{1}{(z+4)(z+2)}$ valid for the region (i) $|z| < 2$ (ii) $2 < |z| < 4$ (iii) $|z| > 4$.	07

OR

	(c)	Determine the Mobius transformation which maps $z_1 = -1, z_2 = i, z_3 = 1$ into $w_1 = 0$, $w_2 = i$, $w_3 = \infty$.Hence, find the image of $|z| < 1$.	07
Q.3	**(a)**	Check whether the function $f(z) = x^2 + y^2 - i2xy$ is analytic or not.	03
	(b)	Evaluate $\oint_C \frac{e^z}{z(z-1)} \, dz$ around the circle $C: |z| = 2$.	04
	(c)	Evaluate the followings: (i) $\int_C \frac{\cos \pi z^2}{(z-2)(z-1)} \, dz$, counter clockwise around the circle $C: |z|$=3. (ii) $\int_C \frac{dz}{z^2+4}$, where C is the unit circle.	07

OR

Q.3	**(a)**	Show that the function $f(z) = \bar{z}$ is nowhere differentiable.	03
	(b)	Expand $f(z) = z^2 \exp\left(\frac{1}{z}\right)$ in Laurent's series about $z = 0$ and classify the singularity.	04
	(c)	Using residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\sin\theta}$.	07
Q.4	**(a)**	Find the value of $\int_0^{1+i} (\bar{z})^2 \, dz$ along the line $y = x$.	03
	(b)	Solve the partial differential equation $(D^2 - 3DD' + 2D'^2)z = \sin(x + 2y)$.	04
	(c)	Obtain the complete integral of the followings: (i) $q - p + x - y = 0$. (ii) $q^2 = z^2 p^2(1 - p^2)$.	07

OR

Q.4	**(a)**	Classify the Partial differential equation $\frac{\partial^2 u}{\partial x^2} + 7 \frac{\partial^2 u}{\partial x \partial y} + 9 \frac{\partial^2 u}{\partial y^2} = 0$.	03

(b) Solve $(x^2 - yz)p + (y^2 - xz)q = z^2 - xy$.   **04**

(c) Find the general solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 25\,\frac{\partial^2 u}{\partial y^2}$ by method of separation of variables.   **07**

Q.5 (a) Solve $px + qy = 3z$.   **03**

(b) Using Charpit's method, solve $q = 3p^2$.   **04**

(c) A tightly stretched string with fixed end points at $x = 0$ and $x = 20$ is initially given by the deflection $f(x) = kx(20 - x)$. If it is released from this position, then find the deflection of the string.   **07**

<div align="center">OR</div>

Q.5 (a) Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$.   **03**

(b) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = y$.   **04**

(c) A rod of 30 cm long has its ends A and B are kept at 20°C and 80°C respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to 0°C and kept so. Find the resulting temperature $u(x, t)$ from the end A.   **07**

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