Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2021

Subject Code:3140610 Date:24/12/2021

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$	**03**
	(b)	Show that the function u=x^2-y^2 is harmonic and find the corresponding analytic function	**04**
	(c)	State DE MOIVRE'S Theorem and Find the fifth root of unity	**07**
Q.2	**(a)**	Prove that $\sinh^{-1} x$ =log(x+$\sqrt{x^2+1}$)	**03**
	(b)	Show that f(z)=$\frac{Re(z^2)}{\lvert z\rvert}$,z$\neq 0$ = 0 ,z=0 Is continuous at z=0.	**04**
	(c)	State Necessary condition for function to be analytic and Show that neither f(z)=$\bar{z}$ nor f(z)=$\lvert z\rvert$ is an analytic function.	**07**
		OR	
	(c)	Using Residue Evaluate $\int_0^\infty \frac{x^2dx}{(x^2+1)(x^2+4)}$.	**07**
Q.3	**(a)**	Evaluate $\int_c (x^2 - iy^2)$ dz along the parabola y=2x^2 from (1,2) to (2,8)	**03**
	(b)	Discuss the convergence of the series. $\sum_{n=0}^{\infty} \frac{(2n)!(z-3i)^n}{(n!)^2}$	**04**
	(c)	State Cauchy Integral Formula and evaluate $\int_c \frac{z-1}{(z+1)^2(z-2)}$ dz,where c is the circle $\lvert z-i\rvert$=2	**07**
		OR	
Q.3	**(a)**	Expand f(z)=$\frac{z-sinz}{z^2}$ at z=0, classify the singular point z=0.	**03**
	(b)	Define Mobius Transformation which maps z=0.i,1 into w=i,-1,∞	**04**
	(c)	Determine the poles of the function f(z)=$\frac{Z^2}{(Z-1)^2(Z+2)}$ and residue at each pole. Hence evaluate $\oint_c f(z)$ where c is the circle $\lvert z\rvert$=3.	**07**
Q.4	**(a)**	Solve z^2(p^{2}z^2+q^2)=1	**03**

	(b)	Solve x(y-z)p+y(z-x)q=z(x-y)	**04**
	(c)	Using method of separation of variable solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$, where $u(x,0)=6e^{-3x}$	**07**

OR

Q.4	**(a)**	Solve $q=3p^2$ by Charpit method	**03**
	(b)	Solve $(D^2+10DD`+25D`^2)Z=e^{3x+2y}$	**04**
	(c)	A tightly stretched string with fixed end points x=0 and x=l is initially in the equilibrium position. If it is set vibrating by giving to each of its points Q, a velocity of $v_0 sin^3\frac{\pi x}{l}$. Find the displacement y(x,t).	**07**
Q.5	**(a)**	Form a partial differential equation by eliminating arbitrary function $xyz=\emptyset(x+y+z)$	**03**
	(b)	Classify the following partial differential equation 1) $2\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+3\frac{\partial^2 u}{\partial y^2}=6$ 2) $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+4\frac{\partial^2 u}{\partial y^2}=0$	**04**
	(c)	Find the solution of wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$, $0\leq x\leq L$ satisfying the condition u(0,t)=u(L,t)=0 $u_t(x,0)=0, u(x,0)=\frac{\pi x}{L}$. $0\leq X\leq L$	**07**

OR

Q.5	**(a)**	Solve $\frac{\partial^2 z}{\partial x^2}=z$	**03**
	(b)	Find the solution of $u_t=c^2u_{xx}$ together with the initial and boundary conditions u(0,t)=u(l,t)=0 for all $t\geq 0$ and u(x,0) $=\sin\frac{\pi x}{l}, \leq x\leq l$	**04**
	(c)	A rod 30 cm long has its end A and B Kept at 20° and 80° c respectively until steady state condition prevail. The temperature at each end is then suddenly reduced to 0° kept so.Find the resulting temperature function u(x,t) from the end A.	**07**

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