Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – SUMMER 2022

Subject Code:3140610 **Date:02-07-2022**

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	(a)	Find an analytic function $f(z)=u+iv$ if $u=x^3-3xy$.	03
	(b)	Find the fourth roots of -1.	04
	(c)	(i) Find the image of infinite strip $0\le x\le 1$ under the transformation $w=iz+1$.	03
		(ii) Separate real and imaginary parts of $f(z)=z^2$.	04
Q.2	(a)	Evaluate $\int_C (x^2+ixy)dz$ from (1, 1) to (2, 4) along the curve $x=t, y=t^2$.	03
	(b)	Determine the mobius transformation that maps $z_1=0, z_2=1, z_3=\infty$ onto $w_1=-1, w_2=-i, w_3=1$ respectively.	04
	(c)	(i) Evaluate $\oint_C \frac{e^z}{z(1-z)^3}dz$, where C is $\lvert z\rvert=\frac{1}{2}$.	03
		(ii) Find the radii of convergence of $\sum_{n=1}^{\infty}\frac{z^n}{2^n+1}$.	04
		OR	
	(c)	Find the image of $\lvert z-1\rvert=1$ under the mapping $w=\frac{1}{z}$.	07
Q.3	(a)	Evaluate $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is the circle $\lvert z\rvert=2$.	03
	(b)	Find $\operatorname{Res}(f(z),4i)$, where $f(z)=\frac{z}{z^2+16}$.	04
	(c)	Expand $f(z)=\frac{1}{(z-1)(z+2)}$ in Laurent's series in the region $(i)\lvert z\rvert<1, (ii) 1<\lvert z\rvert<2, (iii)\lvert z\rvert>2$.	07
		OR	
Q.3	(a)	Evaluate $\oint_C (x^2-y^2+2ixy)dz$, where C is the circle $\lvert z\rvert=1$.	03
	(b)	Evaluate $P.V.\int_{-\infty}^{\infty}\frac{x\cos x}{x^2+9}dx$.	04
	(c)	Find Laurent's series that represent $f(z)=\frac{1}{z(z-1)}$ in the region $(i) 0<\lvert z\rvert<1, (ii) 0<\lvert z-1\rvert<1$.	07

Q.4 **(a)** Solve $\frac{y-z}{yz}p+\frac{z-x}{zx}q=\frac{x-y}{xy}$. **03**

(b) Derive partial differential equation by eliminating arbitrary constants a and b from $z=(x+a)(y+b)$. **04**

(c) (i) Solve $\frac{\partial^3 z}{\partial x^3}=0$. **03**

(ii) Find complete integral of $p^2+q^2=z$. **04**

OR

Q.4 **(a)** Solve $xp+yq=x-y$. **03**

(b) Form a partial differential equation by eliminating arbitrary function from $z=f(x/y)$. **04**

(c) (i) Solve $(D^2-D'^2+D-D')z=0$. **03**

(ii) Solve $q=3p^2$ by Charpit's method. **04**

Q.5 **(a)** Solve $(r+3s+2t)=x+y$ **03**

(b) Solve the p.d.e. $u_{xy}=-u_x$. **04**

(c) Find the deflection $u(x,t)$ of the vibrating string of length π and ends fixed, corresponding to zero velocity and initial deflection $f(x)=k(\sin x-\sin 2x)$. **07**

OR

Q.5 **(a)** Solve $(D^2+DD'+D'-1)z=\sin(x+2y)$. **03**

(b) Solve the p.d.e. $u_x+u_y=2(x+y)u$. **04**

(c) Find the solution of $u_t=c^2u_{xx}$ together with the initial and boundary conditions $u(0,t)=u(l,t)=0$ for all $t\geq 0$ and $u(x,0)=\sin\frac{\pi x}{l}$, $0\leq x\leq l$. **07**

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