Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2023**

**Subject Code:3140610 Date:17-07-2023**

**Subject Name:Complex Variables and Partial Differential Equations**

**Time:10:30 AM TO 01:00 PM Total Marks:70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

| | | | Marks |
|---|---|---|---|
| **Q.1** | **(a)** | Prove that $i^i$ is real. | **03** |
| | **(b)** | Simplify $\frac{(cos5\theta - isin5\theta)^2(cos7\theta + isin7\theta)^{-3}}{(cos4\theta - isin4\theta)^9(cos\theta + isin\theta)^5}$ | **04** |
| | **(c)** | Solve $(D^2 + DD' - 6D'^2)z = \sin(2x + y)$ | **07** |
| **Q.2** | **(a)** | Check whether the function $f(z) = e^{\bar{z}}$ is analytic or not at any point. | **03** |
| | **(b)** | Find and plot all values of $(8i)^{\frac{1}{3}}$. | **04** |
| | **(c)** | Show that the function $u = e^x cosy$ is harmonic. Determine its harmonic conjugate $v(x, y)$ and the analytic function $f(z) = u + iv$. | **07** |
| | | **OR** | |
| | **(c)** | Determine the region in the w-plane into which the triangle bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane is mapped under the transformation $w = 4z$. | **07** |
| **Q.3** | **(a)** | State: i) Cauchy-Goursat Theorem ii)Liouville Theorem. | **03** |
| | **(b)** | Find an upper bound for the absolute value of $\oint_C \frac{e^z}{z+1} dz$ , where C is the circle $\lvert z\rvert = 4$. | **04** |
| | **(c)** | Write Cauchy's Integral formula and hence evaluate $\oint_C \frac{z+1}{z^4+2iz^3} dz$, where C is the circle $\lvert z\rvert = 1$. | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Evaluate $\oint_C \frac{1}{z} dz$, where C is the circle $x = cost, y = sint, 0 \leq t \leq 2\pi$. | **03** |
| | **(b)** | Find the values of $x$ and $y$ if $e^z = \sqrt{3} + i$. | **04** |
| | **(c)** | Evaluate $\int_C \frac{z+1}{z^4-4z^3+4z^2} dz$, where C is the circle $\lvert z - 2 - i\rvert = 2$. | **07** |
| **Q.4** | **(a)** | Identify the type of singularities of $f(z) = \frac{\tan z}{z}$. | **03** |
| | **(b)** | Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$. | **04** |
| | **(c)** | Express $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for the following annular domains. (a) $0 < \lvert z\rvert < 1$ (b) $1 < \lvert z\rvert$ (c) $0 < \lvert z - 1\rvert < 1$ . | **07** |
| | | **OR** | |

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|---|---|---|---|
| **Q.4** | **(a)** | Find the complete integral of $pq = k$, where $k$ is a constant. | **03** |
| | **(b)** | Evaluate by the Residue Theorem $\int_c \frac{1}{(z-1)^2(z-3)} dz$, where the contour C is the rectangle defined by $x = 0, x = 4, y = -1, y = 1$. | **04** |
| | **(c)** | Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$. | **07** |
| **Q.5** | **(a)** | Test for singularity of $\frac{1}{z^2+1}$ and hence find the corresponding residues. | **03** |
| | **(b)** | Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$. | **04** |
| | **(c)** | Using the method of separation of variable, find the solution of $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, given that $u(0, y) = 8e^{-3y}$. | **07** |

**OR**

| | | | |
|---|---|---|---|
| **Q.5** | **(a)** | Solve $(D^3 - 2D^2D')z = 2e^{2x}$. | **03** |
| | **(b)** | Solve $p^2 + q^2 = x + y$. | **04** |
| | **(c)** | Solve $(D^2 + DD' - 6D'^2)z = ycosx$. | **07** |

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