Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2023

Subject Code:3140610 Date: 06-02-2024

Subject Name: Complex Variables and Partial Differential Equations

Time: 10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q. 1 **(a)** 1. State De Moiver's theorem on complex numbers. **01**

2. Write the formula of $\sin h(x+y)$ **01**

3. Find the modulus of $z=\dfrac{1}{\sqrt{3}+i}$ **01**

(b) Prove that $\sinh^{-1} z=\log\left\{z+\sqrt{z^2+1}\right\}$ **04**

(c) Find all the roots of $x^{12}-1=0$ and identify the roots which are also roots of $x^4+x^2+1=0$. **07**

Q. 2 **(a)** Define limit of a complex function. Evaluate $\lim\limits_{z\to i}\dfrac{z-i}{z^2+1}$ **03**

(b) Show that $f(z)=z\,Im(z)$ is differentiable only at $z=0$ and $f'(0)=0$. **04**

(c) State the necessary and sufficient conditions for the complex function $f(z)=u+iv$ to be analytic function. Find the analytic function $f(z)=u+iv$,if $u=x^3-3xy^2$. Also determine its imaginary part v. **07**

OR

(c) Define harmonic function. Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic. Find also its harmonic conjugate v, where $f(z)=u+iv$. **07**

Q. 3 **(a)** State the following on complex analysis: **03**

(i) Cauchy–Goursat theorem (ii) Cauchy's integral formula (iii) Liouville's theorem.

(b) Evaluate $\int_c\left(x^2-iy^2\right)dz$ along the parabola $y=2x^2$ from the points (–2, 8) to (2, 8). **04**

(c) 1. Evaluate $\oint_c \dfrac{e^{2z}}{(z+1)^4}dz$ where c is the circle $|z|=3$. **03**

2. Evaluate $\oint_c \dfrac{\cos\pi z^2}{(z-1)(z-2)}dz$, where c is the circle $|z|=3$. **04**

OR

Q. 3 **(a)** Find the upper bound for $\int_c e^z dz$ where c is the line segment joining the points $(0,0)$ to $(1,2\sqrt{2})$. **03**

	(b)	Find Laurent's series expansion in the power of z that represents the function $f(z)=\dfrac{1}{z^2(1-z)}$ in the domain $\lvert z \rvert > 1$.	04
	(c)	1. Find the radius of convergence of $\sum_{n=0}^{\infty}\dfrac{(2n)!}{(n!)^2}(z-3i)^n$	03
		2. Find residues at each pole for $f(z)=\dfrac{1}{(z-3)(z-1)^2}$	04
Q. 4	(a)	Form the partial differential equation from $z = xy + f(x^2+y^2)$	03
	(b)	Solve $p^2 - q^2 = x - y$	04
	(c)	Solve $(x^2-y^2-z^2)p + 2xy\,q = 2xz$	07
		OR	
Q. 4	(a)	Form the partial differential equation from $(x-a)(y-b)-z^2 = x^2+y^2$.	03
	(b)	Solve $p(1+q) = q\,z$	04
	(c)	Apply Charpit's method to solve $px + qy = pq$	07
Q. 5	(a)	Discuss the classification of the second order linear partial differential equation.	03
	(b)	Solve the equation $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$ by the method of separation of variables given that $u(x,0) = 6\,e^{-3x}$	04
	(c)	Solve $\dfrac{\partial^3 z}{\partial x^3} - 3\dfrac{\partial^3 z}{\partial x^2 \partial y} + 4\dfrac{\partial^3 z}{\partial y^3} = e^{x+2y}$	07
		OR	
Q. 5	(a)	Classify the partial differential equation $\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} = 0$ as parabolic, elliptic or hyperbolic?	03
	(b)	Solve the equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ by the method of separation of variables given that $u(x,0) = \sin \pi x$	04
	(c)	The vibration of an elastic string is governed by the partial differential equation $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$. The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string for t >0.	07

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