Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – WINTER 2021

Subject Code:3130005 **Date:17-02-2022**

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 01:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Represent $z=7i$ into polar form and find the argument of z and the principal value of the argument of z.	**03**
	(b)	State De Moivre's theorem. Find and plot all roots of $(1+i)^{\frac{1}{3}}$ in the complex plane.	**04**
	(c)	Using the method of separation of variables, solve $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ and $u=e^{-5y}$ *when* $x=0$.	**07**
Q.2	**(a)**	Define an analytic function. Write the necessary and sufficient condition for function $f(z)$ to be analytic. Show that $f(z)=\lvert z\rvert^2$ is nowhere analytic.	**03**
	(b)	Define the Mobious transformation. Determine the bilinear transformation which mapping the points $0,\infty,i$ *onto* $\infty,2,0$.	**04**
	(c)	**Attempt the following.**	
		(i) Define the harmonic function. Show that $u=x^2-y^2+x$ is harmonic and find harmonic conjugate of u.	**04**
		(ii) Show that $f(z)=\begin{cases}\frac{\mathrm{Im}(z)}{\lvert z\rvert} & ;z\neq 0\\ 0 & ;z=0\end{cases}$ is not continuous at $z=0$	**03**
		OR	
	(c)	**Attempt the following.**	
		(i) Prove that $\cos^{-1}z=-i\ln(z+i\sqrt{1-z^2})$.	**4**
		(ii) Find the values of $\mathrm{Re}\, f(z)$ and $\mathrm{Im}\, f(z)$ at the point $7+2i$, where $f(z)=\frac{1}{1-z}$.	**3**
Q.3	**(a)**	Evaluate $\int_C \bar{z}dz$, where C is the right- half of the circle $\lvert z\rvert=2$ and hence show that $\int_C \frac{dz}{z}=\pi i$.	**03**
	(b)	Expand $f(z)=\sin z$ in a Taylor series about $z=\frac{\pi}{4}$ and write the Maclaurin series for e^{-z}.	**04**
	(c)	Write the Cauchy integral theorem and Cauchy integral formula and hence evaluate: (i) $\oint_C \frac{e^z}{(z-1)(z-3)}dz; C:\lvert z\rvert=2$. (ii) $\oint_C e^z dz; C:\lvert z\rvert=3$.	**07**

OR

Q.3	**(a)**	Evaluate $\oint_C \frac{e^z}{z+i}dz$, where $c:\|z-1\|=1$.	**03**
	(b)	Develop the following functions into Maclaurin series:(i) $\cos^2 z$ (ii) $e^z \cos z$.	**04**
	(c)	Evaluate $\int_C \mathrm{Re}(z^2)dz$, where C is the boundary of the square with vertices $0, i, 1+i, 1$ in the clockwise direction.	**07**
Q.4	**(a)**	Define the singular points of $f(z)$. Find the singularity of $f(z)$ and classify as pole, essential singularity or removable singularity. where $f(z)=\frac{1-e^z}{z}$.	**03**
	(b)	State the Cauchy residue theorem. Find the residue at its poles of $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ and hence evaluate $\oint_C f(z)dz$, $C:\|z\|=3$.	**04**
	(c)	Determine the Laurent series expansion of $f(z)=\frac{1}{(z+2)(z+4)}$ valid for regions (i) $\|z\|<2$ (ii) $2<\|z\|<4$ (iii) $\|z\|>4$	**07**

OR

Q.4	**(a)**	Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=e^{x+2y}$.	**03**
	(b)	Form a partial differential equation by eliminating the arbitrary functions form the equations $z=f(x+ay)+\phi(x-ay)$.	**04**
	(c)	Show that $\int_{-\infty}^{\infty}\frac{dx}{(x^2+1)(x^2+4)}=\frac{\pi}{6}$.	**07**
Q.5	**(a)**	Form a partial differential equation by eliminating the arbitrary function form $u=f(\frac{x}{y})$.	**03**
	(b)	State the Lagrange's linear partial differential equation of first order and hence solve $x(y-z)p+y(z-x)=z(x-y)$	**04**
	(c)	Using the method of separation of variables, solve $\frac{\partial u}{\partial x}=4\frac{\partial u}{\partial y}$ where $u(0,y)=8e^{-3y}$.	**07**

OR

Q.5	**(a)**	Obtain the general solution of $p+q^2=1$.	**03**
	(b)	Solve by Charpit's method: $px+qy=pq$.	**04**
	(c)	Find the solution of the wave equation $u_{tt}=c^2u_{xx}$, $0\le x\le L$ satisfying the condition $u(0,t)=u(L,t)=0$, $u_t(x,0)=0$, $u(x,0)=\frac{\pi x}{L}$, $0\le x\le L$.	**07**

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