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# GUJARAT TECHNOLOGICAL UNIVERSITY <br> BE - SEMESTER-III (NEW) EXAMINATION - WINTER 2021 <br> Subject Code:3130006 <br> Date:17-02-2022 <br> Subject Name:Probability and Statistics <br> Time:10:30 AM TO 01:00 PM <br> <br> Total Marks:70 <br> <br> Total Marks:70 <br> <br> Instructions: 

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1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.
Q. 1 (a) Define a term random variable and explain different types of random variable.
(b) A card is drawn at random from a pack of 52 cards. What is the probability that the card is a spade or a king?
(c) State Baye's theorem. There are three bags; first containing 1 white, 2 red and 3 green balls; second 2 white, 3 red and 1 green balls and third 3 white, 1 red and 2 green balls. Two balls are drawn from abeg chosen at random. These are found to be 1 white and 1 red. Find the probability that the balls so drawn came from the second bag.
Q. 2 (a) Two judges in a beauty contest rank the 12 contestants as follows:

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 12 | 9 | 6 | 10 | 3 | 5 | 4 | 7 | 6 | 2 | 11 | 1 |

Calculate rank correlation coefficient.
(b) A book contains 100 misprints distributed randomly throughout its 100 pages.

What is the probability that a page observed at random contains at least 2 misprints.
(c) A die is thrown six times. If getting an odd number is a success, find the probability of (i) 5 success (ii) at least five success and (iii) at most five success.

## OR

(c) If a random variable $x$ is Gamma distribution with parameter $\lambda=3$, compute the value of (i) $\mathrm{P}(\mathrm{x} \leq 1)$ and (ii) $\mathrm{P}(1 \leq \mathrm{x} \leq 2)$.
Q. 3 (a) Calculate the coefficient of variance for the following data:

| Class <br> Interval | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 5 | 7 | 2 | 3 | 3 |

(b) Calculate the median for the following data:

| Class <br> Interval | $0-30$ | $30-60$ | $60-90$ | $90-120$ | $120-$ <br> 150 | $150-180$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 8 | 13 | 22 | 27 | 18 | 7 |

(c) Compute the correlation coefficients between X and Y using following data:

| X | 2 | 4 | 5 | 6 | 8 | 11 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 18 | 12 | 10 | 8 | 7 | 5 |
| OR |  |  |  |  |  |  |

Q. 3 (a) Obtain correlation coefficient between x and y if two regression lines are 4x-
$5 y+33=0$ and $20 x-9 y-107=0$.
(b) Calculate the mode for the following data:

| Class <br> Interval | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 10 | 14 | 19 | 7 | 13 |

(c) Obtain the regression line of y on x for the following data:

| x | 100 | 98 | 78 | 85 | 110 | 93 | 80 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 85 | 90 | 70 | 72 | 95 | 81 | 74 |

Q. 4 (a) Explain the term related to testing of hypothesis: (i) Null hypothesis (ii)

Alternate hypothesis and (iii) Errors while accepting or rejecting a hypothesis.
(b) The mean of 35 sample of the thermal conductivity of a certain kind of cement brick is 0.343 with standard deviation of 0.010 . Test the hypothesis that the population mean is 0.340 at $5 \%$ level of significance.
(c) Fit a binomial distribution for the following data showing the survey of 800
families with 4 children and test the goodness of fit.

| No. of boys | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| No. of girls | 4 | 3 | 2 | 1 | 0 |
| No. of families | 32 | 178 | 290 | 238 | 64 |
| OR |  |  |  |  |  |

Q. 4 (a) A random sample of size 15 from bivariate normal distribution gave a correlation coefficient $\mathrm{r}=0.5$. Is this indicate the existence of correlation in the population?
(b) A tire company is suspicious to claim that the average lifetime of certain tires is at least 28000 km . To check the claim, the company takes the sample of 40 tires and gets a mean life time of 27463 km with standard deviation of 1348 km . Test the hypothesis at $1 \%$ level of significance.
(c) Fit a Poisson distribution for the following data and test the goodness of fit.

| x | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| f | $112 \quad$ | 73 | 30 | 4 | 1 |

Q. 5 (a) In $y=a+b x$ if $\sum x=50, \sum y=80, \sum x y=1030, \sum x^{2}=750$ and $n=$ 10 , then find $a$ and $b$.
(b) Fit a curve $y=a e^{b x}$ for the following data:

| x | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| y | 7 | 11 | 17 | 27 |

(c) State properties of the normal distribution. Suppose the marks of 800 students are normally distributed with mean 66 and standard deviation 5. Find number of students getting marks (i) between 65 and 70 (ii) greater than or equal to 72 (Given that $\mathrm{P}(0 \leq \mathrm{z} \leq 0.20)=0.0793$, that $\mathrm{P}(0 \leq \mathrm{z} \leq 0.80)=0.2881$ and that $\mathrm{P}(0 \leq \mathrm{z} \leq 1.2)=0.3849)$

## OR

Q. 5 (a) A random variable x has the following probability distribution:

| $x_{i}$ | 0 | 1 | 2 | 3 |
| ---: | :--- | :--- | :--- | :--- |
| $p_{i}$ | $1 / 6$ | $3 / 8$ | $3 / 8$ | $1 / 8$ |

Find the standard deviation of x for the given distribution.
(b) With usual notations, find the value of p for a binomial random variable x
when $n=6$ and $9 P(x=4)=P(x=2)$.
(c) Fit a parabola $y=a x^{2}+b x+c$ for the following data:

| x | -1 | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| y | -2 | 1 | 2 | 4 |

