GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–III(NEW) EXAMINATION – SUMMER 2023

Subject Code:3130005   **Date:24-07-2023**

Subject Name: Complex Variables and Partial Differential Equations

Time: 02:30 PM TO 05:00 PM   **Total Marks:70**

Instructions:
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

			MARKS
Q.1	**(a)**	Determine an analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$	03
	(b)	Solve the equation $z^2 + (2i - 3)z + 5 - i = 0$	04
	(c)	Show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic and Find a harmonic conjugate of u(x,y)	07
Q.2	**(a)**	Evaluate $\int_C |z|^2 dz$ around the square with vertices at (0,0), (1,0), (1,1), (0,1).	03
	(b)	Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the following regions (i) $|z| < 2$ (ii) $2 < |z| < 4$	04
	(c)	(i) Evaluate $\int_C \frac{zdz}{(z-1)(z-2)}$ where C is the circle $|z| = \frac{1}{2}$	03
		(ii) Evaluate $\int_C \frac{dz}{z^2-7z+12}$ where C is the circle $|z| = 3.5$	04
		OR	
	(c)	Define mobius transformation. Determine the mobius transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ $onto$ $w_1 = -1, w_2 = -i, w_3 = 1$	07
Q.3	**(a)**	Find and plot the image of triangular region in the z-plane with vertices (0,0), (1,0), (0,1) under the transformation $w = (1 - i)z + 3$	03
	(b)	Find the values of a and b such that the function $f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$ is analytic.	04
	(c)	Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and Residue at each pole. Hence evaluate $\int_C f(z)dz$ where C is the circle $|z| = 3$	07
		OR	
Q.3	**(a)**	Expand $f(z) = \frac{1-e^z}{z}$ in Laurent's series about z = 0.	03
	(b)	Find modulus and argument of (i) $\frac{1+2i}{1-(1-i)^2}$ (ii) $\frac{(1+i)^2}{1-i}$	04
	(c)	Evaluate (i) $\int_C \frac{3z^2+7z+1}{z+1} dz$ Where C is $|z| = \frac{1}{2}$	03
		(ii) $\int_C \frac{z^2+1}{z^2-1} dz$ Where C is $|z - 1| = 1$	04
Q.4	**(a)**	Solve $yzp - xzq = xy$	03
	(b)	Form partial differential equation by eliminating the arbitrary constants a and b from $z = axe^y + \frac{1}{2}a^2e^{2y} + b$	04
	(c)	(i) Solve $25r - 40s + 16t = 0$	03
		(ii) Solve $p^2 + q^2 = x + y$	04
		OR	
Q.4	**(a)**	Solve $(mz - ny)p + (nx - lz)q = ly - mx$	**03**

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(b) Form a partial differential equation by eliminating the arbitrary functions from $f(x^2 - y^2, xyz) = 0$ **04**

(c)   (i) Solve $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$ **03**

       (ii) Solve using Charpit's Method $z^2 = pqxy$ **04**

Q.5 **(a)** Solve $\left(D^2 + 10DD' + 25D'^2\right)z = e^{3x+2y}$ **03**

(b) Solve $x\dfrac{\partial u}{\partial x} - 2y\dfrac{\partial u}{\partial y} = 0$ using method of separation of variables.   **04**

(c) (i) Solve $\left(D^2 - D'^2\right)z = x - y$ **03**

     (ii) Solve $\left(2D^2 - 5DD' + 2D'^2\right) = \sin(2x + y)$ **04**

OR

Q.5 **(a)** Solve $(1 - x)p + (z - y)q = 3 - z$ **03**

(b) Solve $2\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial t} + u$ using method of separation of variables subject to the condition $u(x, 0) = 4e^{-3x}$ **04**

(c) Find the solution of the wave equation $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$ such that $y = a\cos pt$ when $x = l$ and $y = 0$ when $x = 0$ **07**

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