

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER– III (New) EXAMINATION – WINTER 2019****Subject Code: 3130908****Date: 26/11/2019****Subject Name: Applied Mathematics for Electrical Engineering****Time: 02:30 PM TO 05:00 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- | | Marks | | | | | | | | | | | | | | |
|---|--------------|---|---|---|---|---|---|---|---|---|---|---|---|---|--|
| Q.1 (a) Find a real root of the equation $x^3 - x - 1 = 0$ by using Regula-falsi method correct to two decimal places. | 03 | | | | | | | | | | | | | | |
| (b) State the formula for finding the q^{th} -root and find the square root of 8 using Newton Raphson method correct to two decimal places. | 04 | | | | | | | | | | | | | | |
| (c) Attempt the following. | | | | | | | | | | | | | | | |
| (i) Find the positive solution of $f(x) = e^{-x} - x$ by the secant method starting from $x_0 = 0, x_1 = 1.0$. | 03 | | | | | | | | | | | | | | |
| (ii) Using method of least squares, find the best fitting straight line to the given following data. | 04 | | | | | | | | | | | | | | |
| <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>x</td> <td>:</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>:</td> <td>1</td> <td>3</td> <td>5</td> <td>6</td> <td>5</td> </tr> </tbody> </table> | x | : | 1 | 2 | 3 | 4 | 5 | y | : | 1 | 3 | 5 | 6 | 5 | |
| x | : | 1 | 2 | 3 | 4 | 5 | | | | | | | | | |
| y | : | 1 | 3 | 5 | 6 | 5 | | | | | | | | | |

- | | | | | | | | | | | | | | | | |
|--|-----------|------|------|-----|-----|------|------|------|------|-----|-----|-----|-----|-----|--|
| Q.2 (a) If $f(x) = \frac{1}{x}$, prepare the table for finite differences and hence find $[a, b]$ and $[a, b, c]$. | 03 | | | | | | | | | | | | | | |
| (b) State Newton's forward formula and use it to find the approximate value of $f(1.6)$, if | 04 | | | | | | | | | | | | | | |
| <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>x</td> <td>1</td> <td>1.4</td> <td>1.8</td> <td>2.2</td> </tr> <tr> <td>f(x)</td> <td>3.49</td> <td>4.82</td> <td>5.96</td> <td>6.5</td> </tr> </tbody> </table> | x | 1 | 1.4 | 1.8 | 2.2 | f(x) | 3.49 | 4.82 | 5.96 | 6.5 | | | | | |
| x | 1 | 1.4 | 1.8 | 2.2 | | | | | | | | | | | |
| f(x) | 3.49 | 4.82 | 5.96 | 6.5 | | | | | | | | | | | |
| (c) Attempt the following. | | | | | | | | | | | | | | | |
| (i) Using quadratic Lagrange interpolation, compute $\ln 9.2$ from $\ln 9.0 = 2.1972$, $\ln 9.5 = 2.2513$, $\ln 11 = 2.3979$ | 03 | | | | | | | | | | | | | | |
| (ii) State Newton's Backward formula and use it to find the approximate value of $f(7.5)$, if | 04 | | | | | | | | | | | | | | |
| <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>x</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>f(x)</td> <td>28</td> <td>65</td> <td>126</td> <td>217</td> <td>344</td> <td>513</td> </tr> </tbody> </table> | x | 3 | 4 | 5 | 6 | 7 | 8 | f(x) | 28 | 65 | 126 | 217 | 344 | 513 | |
| x | 3 | 4 | 5 | 6 | 7 | 8 | | | | | | | | | |
| f(x) | 28 | 65 | 126 | 217 | 344 | 513 | | | | | | | | | |

OR

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|---|-----------|
| (c) Attempt the following. | |
| (i) Using the relation between the operators prove that, $(1 + \Delta)(1 - \nabla) = 1$. | 03 |
| (ii) State Simpson's $\frac{3}{8}$ rule and hence evaluate $\int_0^3 \frac{1}{1+x} dx$ with $n = 6$. | 04 |
| Q.3 (a) Use trapezoidal rule to estimate $\int_{0.5}^{1.3} e^{x^2} dx$ using a strip of width 0.2. | 03 |
| (b) The velocity v of a particle at a distance s from a point on its linear path is given by the following data. | 04 |

Time (t)	0	5	10	15	20	25	30
Speed (v)	30	24	19	16	13	11	10

Estimate the time taken by the particle to travel the distance of 20m using Simpson's $\frac{1}{3}$ rule.

- (c) **Attempt the following.**
- (i) Using Euler's method, find $y(0.2)$ given that **03**
 $\frac{dy}{dx} = y - \frac{2x}{y}; y(0) = 1$ taking $h = 0.1$.
- (ii) State the formula for Runge-Kutta method of fourth order and use **04**
it to calculate $y(0.2)$ given that $y' = x + y, y(0) = 1$ taking $h = 0.1$.

OR

- Q.3 (a)** Define the following. **03**
- 1) Favorable Events
 - 2) Random Variable
 - 3) Probability Density function

- (b) An urn contains 10 white and 3 black balls, while another urn contains **04**
3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the latter. What is the probability that it is a white ball?

- (c) **Attempt the following.**
- (i) In producing screws, let A mean "screw too slim" and B "screw **03**
too small". Let $P(A) = 0.1$ and let the conditional probability that a slim screw is also too small be $P(B/A) = 0.2$. What is the probability that the screw that we pick randomly from a lot produced will be both too slim and too short?

- (ii) The joint probability density function of two random variables X **04**
and Y is given by

$$f(x, y) = \begin{cases} k(x + 2y) & ; 0 < x < 1, 0 < y < 2 \\ 0 & ; elsewhere \end{cases}$$

Find the marginal density function of X and Y.

- Q.4 (a)** Define the following. **03**
- 1) Mutually Exclusive Events
 - 2) Probability
 - 3) Compound Events

- (b) State Bayes' theorem. In a bolt factory, three machines A, B and C **04**
manufacture 25% , 35% and 40% of the total product respectively. Of these outputs 5% , 4% and 2% respectively, are defective bolts. A bolt is picked up at random and found to be defective. What are the probabilities that it was manufactured by machines A, B and C?

- (c) **Attempt the following.**
- (i) A person is known to hit the target in 3 out of 4 shots, where as **03**
another person is known to hit the target in 2 out of 3 shots. Find the probability of the target being hit at all when they both try.
- (ii) Out of five cars, two have tyre problems and one has brake **04**
problem and tow are in good running condition. Two cars are required for the journey. If two cars are selected among five at random and if X denotes the number with tyre problem, Y denotes with brake problem then find the marginal probability function of X and Y.

OR

Q.4 (a) Evaluate $\int_0^1 \exp(-x^2) dx$ by using the Gaussian integration formula for $n = 3$ 03

(b) Using method of least squares, find the best fitting second degree curve to the following data. 04

x	:	1	2	3	4
y	:	6	11	18	27

(c) Attempt the following.

(i) Solve the Ricatti equation $y' = x^2 + y^2$ using Taylor series method for the initial condition $y(0) = 0$, where $0 \leq x \leq 0.2$ and $h = 0.2$. 03

(ii) Find a positive root of the equation $x - \cos x = 0$ using bisection method correct to two places of decimals. 04

Q.5 (a) Define Mean, Median and Mode for the ungrouped data. 03

(b) Find the first four moments about mean $x = 5, 10, 8, 13, 4$ 04

(c) Attempt the following.

(i) In a distribution of two different groups the variances are 15 and 27, whereas the third central moments are 32.4 and 67.56 respectively. Compare the skewness of two groups. 03

(ii) Two automatic filling machines A and B are used to fill mixture of cement concrete in beam. A random sample of beam on each machine showed following results. 04

A	32	28	47	63	71	39	10	60	96	14
B	19	31	48	53	67	90	10	62	40	80

Find standard deviation of each machine and also comment on the performance of the tow machines.

OR

Q.5 (a) The pH solution is measured eight times using the same instrument and the data obtained are as follows. 03

7.15 , 7.20 , 7.18 , 7.19 , 7.21 , 7.20 , 7.16 , 7.18

Calculate the mean, variance and standard deviation.

(b) In environmental geology computer simulation was employed to estimate how far a block from a collapsing rock wall bounce down a soil slope. Based on the depth, location and angle of block soil impact marks left on the slope of the actual rock fall, the following 10 rebound lengths (meters) were estimated. Compute mean and standard deviation of the rebounds. 04

10.2 , 9.5 , 8.3 , 9.7 , 9.5 , 11.1 , 7.8 , 8.8 , 9.5 , 10

(c) Attempt the following.

(i) Find the Co-efficient of Quartile Deviation for the following data: 03
6,8,10,4,20,18,16,14,12,10

(ii) State the formula for coefficient of Skewness based on central moments and finds it for the following frequency distribution. 04

Class	50-55	55-60	60-65	65-70	70-75
Frequency	8	10	15	17	8
