GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER– III (NEW) EXAMINATION – SUMMER 2022

Subject Code:3130006 **Date:11-07-2022**
Subject Name:Probability and Statistics
Time:02:30 PM TO 05:00 PM **Total Marks:70**
Instructions:
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1	**(a)**	Define and give the example of: (i) Random variable, (ii) Independent Events.	03
	(b)	Two fair six-sided dice are tossed independently. Let M be the maximum of the two tosses. What is the probability mass function (pmf) of M?	04
	(c)	Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared. (i) If it has an emergency locator, what is the probability that it will not be discovered? (ii) If it does not have an emergency locator, what is the probability that it will be discovered?	07

Q.2	**(a)**	State the probability function of Exponential and Gamma distribution.	03
	(b)	A dice is thrown 264 times with the following results. Show that the dice is biased. [Use $\chi^2_{0.05} = 11.07$ for 5 degree of freedom].	04

No. appeared on dice	1	2	3	4	5	6
Frequency	40	32	28	58	54	52

	(c)	Fit a straight line to the following data. Also, estimate the value of y at x=72.	07

x:	65	66	67	67	68	69	71	73
y:	67	68	64	68	72	70	69	70

OR

	(c)	Fit the second degree parabola using the least square method to the following data:	07

x:	1	2	3	4	5
y:	5	12	26	60	97

Also, estimate y at x=6.

Q.3	**(a)**	State the properties of the Normal Distribution	03
	(b)	If a random variable has a Poisson distribution such that P(X=1)=P(X=2), find (i) the mean of the distribution, (ii) P(X=5), (iii) P(X>1), and (iv) P(1<X<4).	04
	(c)	Define Binomial Distribution. A particular telephone number is used to receive both voice calls and fax messages. Suppose that 25% of the incoming calls involve fax messages, and consider a sample of 10 incoming calls. What is the probability that (i)At most 3 (ii) Exactly 3 (iii) At least 3 (iv) More than 3, of the calls involve a fax message?	07

OR

Q.3	**(a)**	The mean and variance of a binomial distribution are 4 and 2. Find $P(X \geq 2)$.	03

(b) A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean of 1.5. Calculate the proportion of days on which (i) neither car is used, (ii) some demand is refused, (iii) only one car is used. **04**

(c) Define Standard normal variate. **07**
The lifetime of a certain kind of batteries has a mean life of 400 hours and the standard deviation as 45 hours. Assuming the distribution of lifetime to be normal.
Find The percentage of batteries with lifetime (i) at least 490 hours, (ii) between 385 and 490 hours. Also, find the minimum life of the best 5% of batteries.
[Use: $P(0 < z < 2) = 0.4772$, $P(0 < z < 0.33) = 0.1293$ and $P(0 < z < 1.65) = 0.45$]

Q.4 **(a)** Explain the term related to testing of hypothesis: (i) Type I Error, (ii) Type II Error, (iii) Level of Significance. **03**

(b) A coin was tossed 960 times and returned heads 183 times. Test the hypothesis that the coin is unbiased. Use 5% level of significance. [use $Z_{0.05} = 1.96$]. **04**

(c) Two types of batteries are tested for their length of life and the following data are obtained: **07**

	No. of samples	Mean Life in hours	Variance
Type A	9	600	121
Type B	8	640	144

Is there a significant difference in the two means? [Use $t_{0.05,15} = 2.132$]

OR

Q.4 **(a)** Ten objects are chosen at random from a large population and their weights are found to be in grams:61,63,64,65,68,69,69,70,71,71. Discuss the suggestion that the mean is 65 g. [Use $t_{0.05} = 2.262$ at $v = 9$]. **03**

(b) The means of simple samples of sizes 1000 and 2000 are 67.5 cm and 68 cm respectively. Can the samples be regarded as drawn from the same population of standard deviation 2 cm. [use $Z_{0.05} = 1.96$] **04**

(c) Two random samples are drawn from two populations and the following results were obtained: **07**

Sample I	21	24	25	26	27	
Sample II	22	27	28	30	31	36

Find the variances of the two samples and test whether the two populations have the same variances.[Use $F_{0.05}(5,4) = 6.26$.]

Q.5 **(a)** The probability distribution of a random variable X is given below. **03**
Find (i) E(X), (ii) V(X)

X:	-2	-1	0	1	2
P(x=X)	0.2	0.1	0.3	0.3	0.1

(b) The following are the lines of regression $9y = x + 288$ and $4y = x + 38$. Estimate y when x = 99 and x when y = 30. Also, find the means of x and y. **04**

(c) Ten competitors in a test are ranked by three judges in the following order: **07**

Rank by First Judge:	6	10	2	9	8	1	5	3	4	7
Rank by Second Judge:	5	4	10	1	9	3	8	7	2	6
Rank by Third Judge:	4	8	2	10	7	5	9	1	3	6

Use the method of rank correlation to gauge which pairs of judges has nearest common approach.

OR

Q.5 **(a)** For a group of 10 items, $\Sigma x = 452, \Sigma x^2 = 24270, and\ mode = 43.7$. Find Karl Pearson's coefficient of Skewness. **03**

(b) Find the correlation coefficient for the following data: **04**

X:	-3	-2	-1	0	1	2	3
Y:	9	4	1	0.5	1	4	9

(c) Calculate the regression coefficients and find the two lines of regression for the following data: **07**

x:	57	58	59	59	60	61	62	64
y:	67	68	65	68	72	72	69	71

Find the value of y when x=65.

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