Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019

Subject Code: 3110014 **Date: 06/06/2019**

Subject Name: Mathematics – I

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Use L'Hospital's rule to find the limit of $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{lnx}\right)$.	**03**
	(b)	Define Gamma function and evaluate $\int_0^\infty e^{x^2}dx$.	**04**
	(c)	Evaluate $\int_0^3 \int_{\frac{\sqrt{x}}{3}}^1 e^{y^2}dydx$.	**07**
Q.2	**(a)**	Define the convergence of a sequence (a_n) and verify whether the sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converges or not.	**03**
	(b)	Sketch the region of integration and evaluate the integral $\iint_R (y-2x^2)dA$ where R is the region inside the square $\|x\|+\|y\|=1$.	**04**
	(c)	**(i)** Find the sum of the series $\sum_{n\geq 2}\frac{1}{4^n}$ and $\sum_{n\geq 1}\frac{4}{(4n-3)(4n+1)}$. **(ii)** Use Taylor's series to estimate $sin38°$.	**07**
		OR	
	(c)	Evaluate the integrals $\int_0^\infty\int_0^\infty \frac{1}{(1+x^2+y^2)^2}dxdy$ and $\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{ln3}\frac{\pi e^{2x}sin\pi y^2}{y^2}dxdydz$.	**07**
Q.3	**(a)**	If an electrostatic field E acts on a liquid or a gaseous polar dielectric, the net dipode moment P per unit volume is $P(E)=\frac{e^E+e^{-E}}{e^E-e^{-E}}-\frac{1}{E}$. Show that $\lim_{E\to 0^+}P(E)=0$.	**03**
	(b)	For what values of the constant k does the second derivative test guarantee that $f(x,y)=x^2+kxy+y^2$ will have a saddle point at (0,0)? A local minimum at (0,0)?	**04**
	(c)	Find the series radius and interval of convergence for $\sum_{n=0}^\infty\frac{(3x-2)^n}{n}$. For what values of x does the series converge absolutely?	**07**
		OR	
Q.3	**(a)**	Determine whether the integral $\int_0^3\frac{dx}{x-1}$ converges or diverges.	**03**
	(b)	Find the volume of the solid generated by revolving the region bounded by $y=\sqrt{x}$ and the lines $y=1, x=4$ about the line $y=1$.	**04**

	(c)	Check the convergence of the series $\sum_{n=1}^{\infty}\frac{(lnn)^3}{n^3}$ and $\sum_{n=0}^{\infty}(-1)^n(\sqrt{n+\sqrt{n}}-\sqrt{n})$.	07
Q.4	(a)	Show that the function $f(x,y)=\frac{2x^2y}{x^4+y^2}$ has no limit as (x,y) approaches to (0,0).	03
	(b)	Suppose f is a differentiable function of x and y and $g(u,v)=f(e^u+sinv,e^u+cosv)$. Use the following table to calculate $g_u(0,0), g_v(0,0), g_u(1,2)$ and $g_v(1,2)$.	04

	f	g	f_x	f_y
(0,0)	3	6	4	8
(1,2)	6	3	2	5

	(c)	Find the Fourier series of 2π −periodic function $f(x)=x^2, 0<x<2\pi$ and hence deduce that $\frac{\pi^2}{6}=\sum_{n=0}^{\infty}\frac{1}{n^2}$.	07

OR

Q.4	(a)	Verify that the function $u=e^{-\alpha^2k^2t}\cdot sinkx$ is a solution f the heat conduction euation $u_t=\alpha^2u_{xx}$.	03
	(b)	Find the half-range cosine series of the function $f(x)=\begin{cases}2, & -2<x<0\\ 0, & 0<x<2\end{cases}$.	04
	(c)	Find the points on the sphere $x^2+y^2+z^2=4$ that are closest to and farthest from the point $(3,1,-1)$.	07
Q.5	(a)	Find the directional derivative $D_uf(x,y)$ if $f(x,y)=x^3-3xy+4y^2$ and u is the unit vector given by angle $\theta=\frac{\pi}{6}$. What is $D_uf(1,2)$?	03
	(b)	Find the area of the region bounded y the curves $y=sinx, y=cosx$ and the lines $x=0$ and $x=\frac{\pi}{4}$.	04
	(c)	Prove that $A=\begin{bmatrix}0 & 0 & -2\\ 1 & 2 & 1\\ 1 & 0 & 3\end{bmatrix}$ is diagonalizable and use it to find A^{13}.	07

OR

Q.5	(a)	Define the rank of a matrix and find the rank of the matrix $A=\begin{bmatrix}2 & -1 & 0\\ 4 & 5 & -3\\ 1 & -4 & 7\end{bmatrix}$.	03
	(b)	Use Gauss-Jordan algorithm to solve the system of linear equations $2x_1+2x_2-x_3+x_5=0$ $-x_1-x_2+2x_3-3x_4+x_5=0$ $x_1+x_2-2x_3-x_5=0$ $x_3+x_4+x_5=0$	04
	(c)	State Cayley-Hamilton theorem and verify if for the matrix $A=\begin{bmatrix}4 & 0 & 1\\ -2 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}$.	07