Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I &II (NEW) EXAMINATION – SUMMER-2019

Subject Code: 3110015 **Date: 01/06/2019**

Subject Name: Mathematics –2

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Marks

Q.1 **(a)** Find the Fourier integral representation of **03**

$$f(x) = \begin{cases} x \,; x \in (0, a) \\ 0 \,;\ x \in (a, \infty) \end{cases}$$

(b) Define: Unit step function. Use it to find the Laplace transform of **04**

$$f(t) = \begin{cases} (t-1)^2 \,;\ t \in (0, 1] \\ \ \ 1 \qquad ;\ t \in (1, \infty) \end{cases}$$

(c) Use the method of undetermined coefficients to solve the differential equation $y'' - 2y' + y = x^2 e^x$. **07**

Q.2 **(a)** Evaluate $\oint_C \bar{F} \cdot d\bar{r}$; where $\bar{F} = (x^2 - y^2)\hat{\imath} + 2xy\,\hat{\jmath}$ and $\boldsymbol{C}$ is the curve given by the parametric equation **03**

$\boldsymbol{C} : r(t) = t^2\,\hat{\imath} + t\,\hat{\jmath}\,;\ \ 0 \le t \le 2$.

(b) Apply Green's theorem to find the outward flux of a vector field $\bar{F} = \frac{1}{xy}(x\,\hat{\imath} + y\,\hat{\jmath})$ across the curve bounded by $y = \sqrt{x},\ 2y = 1$ and $x = 1$. **04**

(c) Integrate $f(x, y, z) = x - yz^2$ over the curve $C = C_1 + C_2$, where C_1 is the line segment joining (0,0,1) to (1,1,0) and C_2 is the curve $y=x^2$ joining (1,1,0) to (2,4,0). **07**

OR

(c) Check whether the vector field $\bar{F} = e^{y+2z}\,\hat{\imath} + x\,e^{y+2z}\,\hat{\jmath} + 2x\,e^{y+2z}\,\hat{k}$ is conservative or not. If yes, find the scalar potential function $\varphi(x, y, z)$ such that $\bar{F} =$ grad φ . **07**

Q.3 **(a)** Write a necessary and sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact differential equation. Hence check whether the differential equation **03**

$$[(x+1)e^x - e^y]dx - xe^y dy = 0$$

is exact or not.

(b) Solve the differential equation **04**

$$(1 + y^2)dx = (e^{-\tan^{-1} y} - x)dy$$

(c) By using Laplace transform solve a system of differential equations $\frac{dx}{dt} = 1 - y\,,\ \frac{dy}{dt} = -x\,,$ where $x(0) = 1, y(0) = 0$. **07**

OR

Q.3 **(a)** Solve the differential equation **03**

$$(2x^3 + 4y)dx - xdy = 0.$$

(b) Solve: $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$. **04**

(c) By using Laplace transform solve a differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = e^{-t}$, where $y(0) = 0,\ y'(0) = -1$. **07**

Q.4 **(a)** Find the general solution of the differential equation **03**

$e^{-y}\frac{dy}{dx} + \frac{e^{-y}}{x} = \frac{1}{x^2}$

(b) Solve : $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} + 6y = e^x$ **04**

(c) Find a power series solution of the differential equation $y'' - xy = 0$ near an ordinary point x=0. **07**

OR

Q.4 **(a)** Find the general solution of the differential equation **03**

$\frac{dy}{dx} + \frac{y}{x} - \sqrt{y} = 0.$

(b) Solve : $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = x$ **04**

(c) Find a Frobenius series solution of the differential equation $2x^2y'' + xy' - (x+1)y = 0$ near a regular-singular point x=0. **07**

Q.5 **(a)** Write Legendre's polynomial $P_n(x)$ of degree-n and hence obtain $P_1(x)$ and $P_2(x)$ in powers of x. **03**

(b) Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $y'' + xy' = 0$. **04**

(c) Solve the differential equation **07**

$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3 \cos x$

by using the method of variation of parameters.

OR

Q.5 **(a)** Write Bessel's function $J_p(x)$ of the first kind of order-p and hence show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **03**

(b) Classify ordinary points, singular points, regular-singular points and irregular-singular points (if exist) of the differential equation $xy'' + y' = 0$. **04**

(c) Solve the differential equation $y'' + 25y = \sec 5x$ by using the method of variation of parameters. **07**

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