Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– I & II (NEW) EXAMINATION – WINTER 2019

Subject Code: 3110015 **Date: 01/01/2020**

Subject Name: Mathematics –2

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Find the length of curve of the portion of the circular helix $\vec{r}(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+t\,\hat{k}$ from $t=0$ to $t=\pi$	**03**
	(b)	$\int_{(1,2)}^{(3,4)}\left(xy^2+y^3\right)dx+\left(x^2y+3xy^2\right)dy$ is independent of path joining the points (1, 2) and (3,4). Hence, evaluate the integral.	**04**
	(c)	Verify tangential form of Green's theorem for $\vec{F}=\left(x-\sin y\right)\hat{i}+\left(\cos y\right)\hat{j}$, where C is the boundary of the region bounded by the lines $y=0, x=\pi/2$ and $y=x$.	**07**
Q.2	**(a)**	Find the Laplace transform of $f(t)$ defined as $f(t)=\frac{t}{k}$, $0<t<k$; $=1$, $t>k$	**03**
	(b)	Find the inverse Laplace transform of $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$	**04**
	(c)	(i) Calculate the curl of the vector $xyz\,\hat{i}+3x^2y\,\hat{j}+\left(xz^2-y^2z\right)\hat{k}$ (ii) The temperature at any point in space is given by $T=xy+yz+zx$. Determine the derivative of T in the direction of the vector $3\hat{i}-4\hat{k}$ at the point (1, 1, 1).	**07**
		OR	
	(c)	Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, $r=\left|\vec{r}\right|$, and $\bar{a}$ is a constant vector. Find the value of $div\left(\frac{\bar{a}\times\bar{r}}{r^n}\right)$	**07**
Q.3	**(a)**	Find constants a, b and c such that $\vec{V}=\left(x+2y+az\right)\hat{i}+\left(bx-3y-z\right)\hat{j}+\left(4x+cy+2z\right)\hat{k}$ is irrotational.	**03**
	(b)	Using Fourier cosine integral representation show that $\int_0^\infty \frac{\cos\omega x}{k^2+\omega^2}d\omega=\frac{\pi e^{-kx}}{2k}$	**04**
	(c)	Solve the following differential equations: (i) $\cos(x+y)\,dy=dx$ (ii) $\sec^2 y\frac{dy}{dx}+x\tan y=x^3$	**07**
		OR	

Q.3	**(a)**	Find the Laplace transform of (i) $\int_0^t \frac{\sin t}{t} dt$ (ii) $t^2 u(t-3)$	**03**
	(b)	Using Convolution theorem obtain $L^{-1}\left(\frac{1}{s(s^2+a^2)}\right)$	**04**
	(c)	Find the power series solution of $\frac{d^2y}{dx^2}+xy=0$	**07**
Q.4	**(a)**	Find the Laplace transform of the waveform $f(t)=\left(\frac{2t}{3}\right), 0 \le t \le 3$	**03**
	(b)	Using the Laplace transforms, find the solution of the initial value problem $y''+25y=10\cos 5t \quad y(0)=2, y'(0)=0$	**04**
	(c)	Using variation of parameter method solve $(D^2+1)y = x\sin x$	**07**

OR

Q.4	**(a)**	Solve $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\frac{dy}{dx}$	**03**
	(b)	Solve $y'''-3y''+3y'-y=4e^t$	**04**
	(c)	Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+4y=2x^2+3e^{-x}$ using method of undetermined coefficients.	**07**
Q.5	**(a)**	Classify the singular points of the equation $x^3(x-2)y''+x^3y'+6y=0$	**03**
	(b)	Solve $(D^2+4)y=\cos 2x$	**04**
	(c)	Solve (i) $ye^x dx+(2y+e^x)dy=0$ (ii) $\frac{dy}{dx}+2y\tan x=\sin x$	**07**

OR

Q.5	**(a)**	Solve $\frac{dy}{dx}=\frac{y^3}{e^{2x}+y^2}$	**03**
	(b)	If $y_1 = x$ is one of solution of $x^2y''+xy'-y=0$ find the second solution.	**04**
	(c)	Using Frobenius method solve $x^2y''+4xy'+(x^2+2)y=0$	**07**

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