Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE – SEMESTER 1&2 EXAMINATION – SUMMER 2020

Subject Code: 3110015 **Date:09/11/2020**

Subject Name: Mathematics II

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Evaluate $\int_c \bar{F}\cdot d\bar{r}$ along the parabola $y^2 = x$ between the points (0, 0) and (1, 1) where $\bar{F} = x^2\hat{i} + xy\,\hat{j}$	**03**
	(b)	Find the work done in moving particle from A (1, 0, 1) to B (2,1,2) along the straight-line AB in the force field $\bar{F} = x^2\hat{i} + (x-y)\,\hat{j} + (y+z)\,\hat{k}$	**04**
	(c)	Verify green`s theorem for $\oint_c (2xydx - y^2dy)$ where C is the boundary of the region bounded by the ellipse $3x^2 + 4y^2 = 12$	**07**
Q.2	**(a)**	Find the Laplace transform of $te^{-4t}\sin 3t$.	**03**
	(b)	Find the inverse Laplace transform of $\dfrac{5s+3}{(s-1)(s^2+2s+5)}$.	**04**
	(c)	Show that the vector field $\bar{F} = (y\sin z - \sin x)\hat{i} + (x\sin z + 2yz)\,\hat{j} + (xy\cos z + y^2)\,\hat{k}$ is conservative and find the corresponding scalar potential.	**07**
		OR	
	(c)	Show that $\bar{F} = 2xyz\,\hat{i} + (x^2z + 2y)\,\hat{j} + x^2y\,\hat{k}$ is irrotational and find a scalar function ϕ such that $\bar{F} = grad\phi$.	**07**
Q.3	**(a)**	Find the directional derivative of $f(x,y) = xy + xe^y + \cos(xy)$ at the point $P(1,0)$ in the direction of $\bar{u} = 3\hat{i} - 4\hat{j}$.	**03**
	(b)	Find the inverse Laplace transform of $\log\left(1+\dfrac{1}{s^2}\right)$.	**04**
	(c)	Find the singular solution and general solution of $y + px = x^4p^2$	**07**
		OR	
Q.3	**(a)**	Find the Laplace transform of $\dfrac{\cos at - \cos bt}{t}$.	**03**
	(b)	Show that $\int_0^\infty \dfrac{\omega^3 \sin\omega x}{\omega^4+4}\,d\omega = \dfrac{\pi}{2}e^{-x}\cos x; x>0.$	**04**
	(c)	Find the power series solution of $y' - 2xy = 0;\ y(0) = 1$ near $x = 0$.	**07**

Q.4	**(a)**	Find the Laplace transform of $e^{-t}\left\{1-u\left(t-2\right)\right\}$.	**03**
	(b)	Solve $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t$ with $x=2, \frac{dx}{dt}=-1$ at $t=0$.	**04**
	(c)	Solve $\left(D^2-1\right)y=xe^x\sin x$	**07**

OR

Q.4	**(a)**	Solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$	**03**
	(b)	Using method of variation of parameter, solve $\frac{d^2y}{dx^2}+4y=\tan 2x$.	**04**
	(c)	Using method of undetermined coefficients solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^x$.	**07**
Q.5	**(a)**	Classify the singular points of $x^2y''+xy'-2y=0$.	**03**
	(b)	Solve $\frac{d^2y}{dx^2}+9y=\sin 2x\sin x$.	**04**
	(c)	Solve (i) $\left(x^3+3xy^2\right)dx+\left(3x^2y+y^3\right)dy=0$. (ii) $\frac{dy}{dx}+y\cot x=2\cos x$.	**07**

OR

Q.5	**(a)**	Solve $\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}$.	**03**
	(b)	Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+y=\cos\left(\ln x\right)$.	**04**
	(c)	Using Frobenius method solve $2x^2y''+xy'-\left(x+1\right)y=0$.	**07**

****