Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2021

Subject Code:3110014 **Date:19/03/2022**

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	If $u = \log(tanx + tany + tanz)$ then show that $$sin2x\frac{\partial u}{\partial x} + sin2y\frac{\partial u}{\partial y} + sin2z\frac{\partial u}{\partial z} = 2$$	**03**
	(b)	Evaluate $$\lim_{x\to 0}\frac{e^x - e^{-x} - 2log(1+x)}{xsinx}$$	**04**
	(c)	Find the extreme values of the function $$f(x,y) = x^3 + y^3 - 3x - 12y + 20$$	**07**
Q.2	**(a)**	Use Ratio test to check the convergence of the series $$\sum_{n=1}^{\infty}\frac{2^n+1}{3^n+1}$$	**03**
	(b)	Find the Maclaurin's series of $cosx$ and use it to find the series of sin^2x.	**04**
	(c)	Find the Fourier series of $f(x) = x^2$ in the interval $(0,2\pi)$ and hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$	**07**
		OR	
	(c)	Find the Fourier series of $f(x) = 2x - x^2$ in the interval (0,3).	**07**
Q.3	**(a)**	Find the directional derivative of $f(x,y,z) = xyz$ at the point $P(-1,1,3)$ in the direction of the vector $\bar{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.	**03**
	(b)	Find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to row echelon form.	**04**
	(c)	Find the eigenvalues and corresponding eigenvectors of the matrix $$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$	**07**
		OR	
Q.3	**(a)**	If $u = f(x-y, y-z, z-x)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	**03**
	(b)	Find the inverse of the following matrix by Gauss-Jordan method:	**04**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

	(c)	Verify Cayley-Hamilton theorem for the following matrix and use it to find A^{-1} $$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$	**07**
Q.4	**(a)**	If 1 is an eigenvalue of the matrix $\begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ then find its corresponding eigen vector.	**03**
	(b)	Expand $2x^3 + 7x^2 + x - 1$ in powers of $(x - 2)$	**04**
	(c)	Solve following system by using Gauss Jordan method $$x + 2y + z - w = -2$$ $$2x + 3y - z + 2w = 7$$ $$x + y + 3z - 2w = -6$$ $$x + y + z + w = 2$$	**07**

OR

Q.4	**(a)**	Use integral test to show that the following infinite series is convergent $$\sum_{n=1}^{\infty} \frac{1}{n(1 + \log^2 n)}$$	**03**
	(b)	For the odd periodic function defined below, find the Fourier series $$f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$$	**04**
	(c)	Determine the radius and interval of convergence of the following infinite series $$x - \frac{x^2}{4} + \frac{x^3}{9} - \frac{x^4}{16} + \cdots$$	**07**
Q.5	**(a)**	Show the following limit does not exist using different path approach $$\lim_{(x,y)\to(0,0)} \frac{2x^2y^2}{x^4 + y^4}$$	**03**
	(b)	Evaluate the following integral along the region R $$\iint_R (x + y)\,dydx$$ where R is the region bounded by $x = 0, x = 2, y = x, y = x + 2$. Also, sketch the region.	**04**
	(c)	Change the order of integration and hence evaluate the same. Do sketch the region. $$\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\,dydx$$	**07**

OR

Q.5	**(a)**	The following integral is an improper integral of which type? Evaluate $$\int_0^{\infty} \frac{dx}{x^2 + 1}$$	**03**

(b) If $x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi, z = r\cos\theta$, then find the jacobian **04**

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)}$$

(c) Find the volume of the solid generated by rotating the region bounded by $y = x^2 - 2x$ and $y = x$ about the line $y = 4$. **07**

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