

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-I & II(NEW)EXAMINATION – SUMMER 2022****Subject Code:3110014****Date:02-08-2022****Subject Name:Mathematics - 1****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

	Marks
Q.1 (a) Is $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent for $p > 1$? Justify your answer.	03
(b) (1) Find $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{(x-a)^2}$	02
(2) Is $\int_0^{\infty} \frac{\sin^2 x}{x^2}$ convergent? Justify your answer.	02
(c) (1) Find the length of curve $f(x) = \frac{x^3}{12} + \frac{1}{x}, 1 \leq x \leq 4.$	04
(2) Prove that $\Gamma(n) = (n-1) \Gamma(n-1).$	03
Q.2 (a) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{n^2}{7^n}.$	03
(b) Investigate the convergence of $\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$	04
(c) Find Fourier series of $f(x) = x^2, -\pi < x < \pi.$	07
OR	
(c) Find Fourier series of $f(x) = x, -1 < x < 0$ $= 2, 0 < x < 1.$	07
Q.3 (a) Find the derivative of $f(x, y) = x^2 + xy + y^2$ in the direction $\hat{i} + \hat{j}$ at $P(1,1).$	03
(b) Find the tangent plane of $z = e^x \cos y$ at $P(0,0,0).$	04
(c) Find local extreme values of $f(x, y) = xy - x^2 - y^2 - x.$	07
OR	
Q.3 (a) Explain second derivative test for local extreme values.	03
(b) Let $f = \ln r$, where $\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$ and $r = \vec{r} .$ Find $\text{grad } f.$	04
(c) Determine the minimum value of $x^2 y z^2$ subject to the condition $x + y + 2z = 5$ using method of Lagrange multipliers.	07
Q.4 (a) Evaluate $\int_{y=0}^1 \int_{x=0}^2 \frac{1}{\sqrt{4-x^2} \sqrt{1-y^2}} dx dy.$	03

(b) Evaluate the integral $\int_0^2 \int_{x/2}^1 \frac{1}{3} e^{y^2} dy dx$ 04

by change of order.

(c) (1) Find the area of the region covered by $x = 1$, $x = 4$, $y = 0$ and $y = \sqrt{x}$. 04

(2) Evaluate $\int_{x=0}^1 \int_{y=0}^{x^{1/4}} \int_{z=0}^{y^2} \sqrt{z} dz dy dx$ 03

OR

Q.4 (a) Evaluate $\iint_R xy dA$ where R is the region 03

bounded by x axis, the ordinate $x = 2a$ and the curve $x^2 = 4ay$.

(b) Evaluate the integral $\int_{y=0}^1 \int_{x=0}^{\cos^{-1}y} e^{\sin x} dx dy$ by change of order. 04

(c) (1) Change in to polar coordinates then solve $\int_0^2 \int_0^{\sqrt{4-x^2}} e^{-(x^2+y^2)} dy dx$. 04

(2) Let $x + y = u$ and $y = uv$ are given transformations. Find Jacobian for change of variables. 03

Q.5 (a) Find characteristic equation of $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$. 03

(b) Find Maclaurin's series for $f(x) = e^{2x} \sinh x$ and show at least up to x^4 term. 04

(c) Solve $x + y + w = 1$, $2x + z + w = 3$, $2y + z + 2w = 2$. 07

OR

Q.5 (a) Show that give matrix $A = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$ satisfies its Characteristic equation. 03

(b) Show that $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges. 04

(c) Show that $A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -1 & 2 \\ -4 & -8 & 7 \end{bmatrix}$ is diagonalizable. Find the matrix of eigen vectors and diagonal matrix. 07

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