Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I & II(NEW) EXAMINATION – WINTER 2022

Subject Code:3110014 Date:02-03-2023

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** Find $\lim_{x\to 0} \frac{tan^2x-x^2}{x^2tan^2x}$. **03**

(b) If $z = tan^{-1}\frac{x}{y}$, $x = u\cos v$ and $y = u\sin v$ then find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule. **04**

(c) Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3xy$. **07**

Q.2 **(a)** Find the directional derivative of the function $f(x,y) = x^2 \sin 2y$ at the point $\left(1,\frac{\pi}{2}\right)$ in the direction of $v = 3\hat{\imath} - 4\hat{\jmath}$ **03**

(b) Show that the series $\sum_{n=0}^{\infty}\left(\frac{e}{\pi}\right)^n$ is convergent and find its sum. **04**

(c) Find the Fourier series of the 2π periodic function $f(x) = x + |x|,\ -\pi < x \le \pi$. **07**

OR

(c) Find the half range sine series of the function $f(x) = x - x^2$ in the interval $(0,1)$ and hence determine the sum $\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}$. **07**

Q.3 **(a)** Test the convergence of the series $\sum_{n=1}^{\infty}\frac{2n+1}{n^2+n+1}$. **03**

(b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$. **04**

(c) Solve the following system of linear equations using the Gauss-Jordan Method. **07**

$$x_1 + x_2 + 2x_3 = 8, -x_1 - 2x_2 + 3x_3 = 1, 3x_1 - 7x_2 + 4x_3 = 10.$$

OR

Q.3 **(a)** Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(n+1)\sin\left(\frac{(2n-1)\pi}{2}\right)}{n\log(n+1)}$ **03**

(b) Find the approximate value of $\sqrt{25.15}$ using Taylor's theorem. **04**

(c) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$. **07**

Q.4 **(a)** Let $A = \begin{bmatrix} -3 & -1 & 1 \\ 0 & 2 & 3 \\ 1 & -2 & 1 \end{bmatrix}$. Using Cayley-Hamilton theorem, find the matrix A^3. **03**

(b) Find the equations of the tangent plane and normal line to the surface $x^2yz + 3y^2 = 2xz^2 - 8z\ at\ P(1,2,-1)$. **04**

(c) Find the length of the arc of the curve $y = \log\left(\frac{e^x - 1}{e^x + 1}\right)$ from $x = 1\ to\ x = 2.$ **07**

OR

Q.4 **(a)** Show that $f(x,y) = \begin{cases} \frac{x^3 y}{x^4 + y^4}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}$ is not continuous at$(0,0)$. **03**

(b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ using Gauss-Jordan method. **04**

(c) Sketch the region R bounded by the lines $x = 0, x = 2,\ x = y$ and $y = x + 2$. Find the area of this region R by double integrals. **07**

Q.5 **(a)** Evaluate $\int_0^1 \int_x^1 \int_0^{y-x} dz\, dy\, dx.$ **03**

(b) Evaluate $\iint_R y\,(x - y^2) dydx$ **04**

where R is the region bounded by the curves $y = \sqrt{x}$ and $y = x^3$.

(c) Determine all the positive values of x for which the series **07**

$$\frac{1}{1\cdot 2\cdot 3} + \frac{x}{4\cdot 5\cdot 6} + \frac{x^2}{7\cdot 8\cdot 9} + \cdots\cdots\cdots$$

converges.

OR

Q.5 **(a)** Evaluate $\iint_R xy\sqrt{x^2 + y^2}\, dxdy$ by changing into polar coordinates where $R = \{(x,y): 1 \le x^2 + y^2 \le 4, x \ge 0, y \ge 0\}$. **03**

(b) Discuss the convergence of the improper integral $\int_1^\infty \frac{\log x}{x^2} dx$. **04**

(c) Evaluate $\int_0^1 \int_{1-\sqrt{1-y^2}}^{1+\sqrt{1-y^2}} dx\, dy$ by changing the order of integration. Give the sketch of region of integration. **07**