

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-I & II(NEW) EXAMINATION – SUMMER 2023****Subject Code:3110014****Date:31-07-2023****Subject Name:Mathematics - 1****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

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|------------|---|-----------|
| Q.1 | (a) Evaluate: $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$. | 03 |
| | (b) Show that series $\sum_{n=0}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$ is convergent. Hence find sum of the series. | 04 |
| | (c) Define eigenvalue and eigen vector of a matrix. Find all eigenvalues and eigenvectors of matrix
$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$. | 07 |
| Q.2 | (a) Evaluate: $\int_0^1 \int_x^{2x} \int_0^y 2xyz \, dz \, dy \, dx$. | 03 |
| | (b) Find equations of tangent plane and normal line to the surface $\frac{x^2}{2} - \frac{y^2}{3} - z = 0$ at $(2, 3, -1)$. | 04 |
| | (c) Find Fourier series of 2π -periodic function
$f(x) = x + x^2; -\pi < x < \pi$. | 07 |
| OR | | |
| | (c) Find the Fourier series for the periodic extension of
$f(x) = \begin{cases} \sin x, & 0 \leq x \leq \pi \\ 0, & \pi \leq x \leq 2\pi \end{cases}$. | 07 |
| Q.3 | (a) Define Improper integrals of type-I. Evaluate $\int_{-1}^{\infty} \frac{dt}{t^2 + 5t + 6}$. | 03 |
| | (b) Find the volume of the solid generated by revolving the region bounded by $y = x^2, y = 0$ and $x = 2$ about the x -axis. | 04 |
| | (c) (i) Using ratio test discuss convergence of series $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$. | 03 |
| | (ii) Use Taylor series to represent function
$f(x) = 2x^3 + x^2 + 3x - 8$ in powers of $(x-1)$. | 04 |

OR

- Q.3 (a)** Define Beta function. Evaluate $\int_0^1 x^4(1-\sqrt{x})^5 dx$. **03**
- (b)** Find the arc length of $f(x) = \ln(\cos x)$, $0 \leq x \leq \pi/4$. **04**
- (c) (i)** Using Sandwich theorem find limit of sequence **03**
 $(an) = \left(\frac{\cos n}{n}\right)$.
- (ii)** Define Radius of Convergence of power series. Find it for **04**
power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 3^n}$.
- Q.4 (a)** Find the sum of series $\sum_{n=0}^{\infty} \frac{2^n - 1}{3^n}$. **03**
- (b)** If $u = \log(x^2 + y^2)$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. **04**
- (c)** Find all local maxima, local minima and saddle points of the **07**
function $f(x, y) = x^3 + 3xy^2 - 15x + y^3 - 15y$.
- OR**
- Q.4 (a)** Discuss Convergence of series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$. **03**
- (b)** Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^2}{x^4 + y^2}$ does not exist. **04**
- (c)** A rectangular box without a lid is to be made from 12 m² of **07**
cardboard. Find the maximum volume of such a box.
- Q.5 (a)** Find Jacobian $\frac{\partial(x, y)}{\partial(r, \theta)}$ for $x = r \cos \theta$, $y = r \sin \theta$. **03**
- (b)** Solve the given linear system using Gauss-Elimination method **04**
 $x_1 + x_2 - 2x_3 = 1$, $2x_1 - 3x_2 + x_3 = -8$, $3x_1 + x_2 + 4x_3 = 7$.
- (c)** Sketch the region of integration, change the order of integration **07**
and evaluate $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3 + 1} dy dx$.
- OR**
- Q.5 (a)** Evaluate: $\int_0^2 \int_0^{\pi/2} x \sin y dy dx$. **03**
- (b)** State Cayley-Hamilton theorem. Verify it for matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$. **04**
- (c)** Use the transformation $x = u^2 - v^2$, $y = 2uv$ to evaluate the **07**
integral $\int_0^1 \int_0^{2\sqrt{1-x}} \sqrt{x^2 + y^2} dy dx$.
