GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I & II(NEW) EXAMINATION – SUMMER 2023

Subject Code:3110015   **Date:07-08-2023**

Subject Name:Mathematics - 2

Time:10:30 AM TO 01:30 PM   **Total Marks:70**

Instructions:
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

			Marks
Q.1	**(a)**	Find the directional derivative of $f(x,y,z) = x^2 + 5y^2 + 3z^2$ at point $(1,1,1)$ in the direction 3i+4j+5k.	03
	(b)	A vector field is given by $F = 3x^2yz\, i + x^3z\, j + x^3y\, k$, show that f is irrotational and find the scalar $\emptyset$ such that $F = \nabla\emptyset$.	04
	(c)	Verify the Greens theorem for $F = x^2 i + xy^2\, j$, along the square bounded by $x = 0,\quad x = 1,\quad y = 0\ and\ y = 1$	07
Q.2	**(a)**	Discuss about ordinary point, singular point, regular singular point and irregular singular point of differential equation $(x^2 - 1)y'' + 3xy + 5x^2 y = 0$	03
	(b)	Express $f(x) = x^3 + 2x^2 + 3x - 1$ in terms of Legendre's polynomial.	04
	(c)	Find a power series solution of $y'' - 25y = 0$ near an ordinary point x=0.	07

OR

	(c)	Prove that $Pn(x) = \dfrac{1}{n!2^n}\dfrac{d^n}{dx^n}(x^2 - 1)^n$	07
Q.3	**(a)**	Find the Laplace transform of (i) $e^{3t}t^5$ (ii) t cos3t (iii)($\sin^2 5t$)/t	03
	(b)	Find the Inverse Laplace transform of (i) $\dfrac{1}{(s-1)(s-2)(s-3)}$ (ii) $\dfrac{1}{s^4-9s^2}$ (iii) $\dfrac{s+1}{s^2+2s+10}$ (iv) $tan^{-1}(\dfrac{s}{4})$	04
	(c)	Solve the initial value problem using Laplace transformation $y'' - 5y' - 6y = e^{3t}$ with $$y(0) = 3\ and\ y'(0) = 2$$	07

OR

Q.3	**(a)**	Find the Laplace transform of (i) $e^{4t}u(t - 2)$ (ii) $(t^2 + 1)u(t - 1)$ (iii) $sin3t\ u(t - \pi)$	03
	(b)	Using convolution theorem find the Inverse Laplace transform of $\dfrac{1}{(s^2 -4)(s^2 -9)}$	04
	(c)	Find the Fourier integral of	07

$$f(x) = \begin{cases} 0, & |x| < 5 \\ 5, & |x| > 5 \end{cases}$$

Q.4 **(a)** Solve the Differential equation   03
$e^x cosy \, dx - e^x siny \, dy = 0$

(b) Solve $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$, $y(0) = 1$, $y'(0) = 2$ 04

(c) Solve   07

     (i) $\frac{dy}{dx} + y \, tanx = sin2x$, $y(0) = 2$

     (ii) $(x^3 + y^3)dx - xy^2 dy = 0$

<div align="center">OR</div>

Q.4 **(a)** Solve $(x^2 - y^2)dx + xydy = 0$ 03

(b) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 14y = 0$, $y(0) = 3$, $y'(0) = 1$ 04

(c) Solve   07

     (i) $(y - px)(p^2 + 1) = tan^{-1}p$

     (ii) $p^2 x^2 = x^2 + p^2$

Q.5 **(a)** Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = e^{-4x} - 12x$ 03

(b) Using method of undetermined coefficient obtain the solution of   04
$\frac{d^2y}{dx^2} + 4y = sinx$

(c) Find the solution of differential equations   07

(1) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x^2 e^x$

(ii) $\frac{x^2 d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = cos \, (logx)$

<div align="center">OR</div>

Q.5 **(a)** Solve   03

$$\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = cos2x$$

(b) Using method of undetermined coefficient   04
$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = x^2$

(c) (i) Using method of variation of parameters find the solution of   07
differential equation $\frac{d^2y}{dx^2} + 16y = cot4x$

(ii) Find the solution of differential equation
$$\frac{x^2 d^2y}{dx^2} - 6x\frac{dy}{dx} + 6y = x^2 + \frac{1}{x^2}$$

<div align="center">*************************</div>