

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER- III(NEW) EXAMINATION – WINTER 2022

Subject Code:3130006

Date:20-02-2023

Subject Name:Probability and Statistics

Time:02:30 PM TO 05:00 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- | | | Marks | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|--------------------|--|-----------|---|---|---|---|---|---|---|----|---|---|--------------------|---|----|---|---|---|---|---|---|---|---|----------------|---|---|---|---|---|---|---|---|---|----|--|
| Q.1 | (a) Define random variable and continuous random variable. If X be a continuous random variable with a probability density function $f(x) = kx(1-x)$, $0 \leq x \leq 1$ find k . | 03 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | (b) Define Mutually exclusive events and independent events. If A and B are independent events, where $P(A)=1/4$, $P(B)=2/3$. Find $P(A \cup B)$. | 04 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | (c) State Baye's theorem. An urn I contains 3 white and 4 red balls and an urn II contains 5 white and 6 red balls. One ball is drawn at random from one of the urns and is found to be white. Find the probability that it was drawn from urn I. | 07 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q.2 | (a) Compute Spearman's rank correlation coefficient for the following data: | 03 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1" style="border-collapse: collapse; width: 100%; text-align: center;"> <thead> <tr> <th>Person</th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> <th>H</th> <th>I</th> <th>J</th> </tr> </thead> <tbody> <tr> <td>Rank in statistics</td> <td>9</td> <td>10</td> <td>6</td> <td>5</td> <td>7</td> <td>2</td> <td>4</td> <td>8</td> <td>1</td> <td>3</td> </tr> <tr> <td>Rank in income</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> </tr> </tbody> </table> | Person | A | B | C | D | E | F | G | H | I | J | Rank in statistics | 9 | 10 | 6 | 5 | 7 | 2 | 4 | 8 | 1 | 3 | Rank in income | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| Person | A | B | C | D | E | F | G | H | I | J | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rank in statistics | 9 | 10 | 6 | 5 | 7 | 2 | 4 | 8 | 1 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rank in income | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | |
| | (b) A manufacturer knows from experience that the resistance of resistors he produces is normal with a mean $\mu = 100$ ohms and standard deviation $\sigma = 2$ ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms? $P(0 < z < 1) = 0.3413$. | 04 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | (c) In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets. ($e^{-0.02} = 0.9802$) | 07 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | OR | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | (c) Define exponential distribution. The average time it takes to serve a customer at a petrol pump is 6 minutes. The service time follows exponential distribution. Calculate the probability that (i) A customer will take less than 2 minutes to complete the service. (ii) A customer will take between 4 and 5 minutes to get the service. (iii) A customer will take more than 10 minutes for his service. | 07 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Q.3 | (a) Obtain the correlation coefficient between x and y , when the lines of regression are: $2x-9y+6 = 0$, $x-2y+1 = 0$. | 03 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

- (b) Calculate the mode from the following data:

Age	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Frequency	6	11	25	35	18	12	6

04

- (c) The following table gives recorded data showing the test scores made by salesmen on an intelligence test and their weekly sales:

Salesmen	1	2	3	4	5	6	7	8	9	10
Test scores	40	70	50	60	80	50	90	40	60	60
Sales	2.5	6.0	4.5	5.0	4.5	2.0	5.5	3.0	4.5	3.0

07

Calculate the regression line of sales on test scores and estimate the most probable weekly sales volume if a salesman makes a score of 70.

OR

- Q.3 (a) In a distribution, the mean=65, median =70, coefficient of skewness =-0.6. Calculate the mode and coefficient of variation.

03

- (b) Calculate the median for the following data:

04

Class(Marks group)	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	5	6	15	10	5	4	2	2

- (c) Following are the values of import of raw material and export of finished product in suitable units.

07

Export	10	11	14	14	20	22	16	12	15	13
Import	12	14	15	16	21	26	21	15	16	14

Compute the coefficient of correlation between the import values and export values.

- Q.4 (a) Explain the term related to testing of hypothesis (i) Null hypothesis (ii) Level of significance (iii) Two tailed test.

03

- (b) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain exactly two defective parts?

04

- (c) The following table showing the number of plants having certain characters.

07

	Flat leaves	Curled leaves	Total
White flowers	267	27	294
Red flowers	757	155	912
Total	1024	182	1206

Use the χ^2 test to access the correctness of the hypothesis that the flower colour is independent of flatness of leaf at 5% level of significance. $\chi^2_{(0.05,1)} = 3.84$.

OR

- Q.4 (a) The mean life of a sample of 25 fluorescent light bulbs is found as 1550 hours with a standard deviation of 120 hours. The company manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptance at 5% level of significance? The value of t at 5% level of significance and 24 degree of freedom is 1.711.

03

- (b) The mean life time of sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. The company claims that the average life of the bulbs produced by it is 1600 hours. Using the level of significance of 0.05, is the claim acceptable? ($Z_{0.05} = \pm 1.96$).

04

- (c) State Chebyshev's inequality. A fair dice is tossed 120 times. Use Chebyshev's inequality to find a lower bound for the probability of getting 80 to 120 sixes. 07

- Q.5** (a) Fit the line $y = a + bx$ for the following data: 03

x	0	5	10	15	20	25
y	12	15	17	22	24	30

- (b) Fit the curve $y = ax^b$ to the following data. 04

x	61	26	7	2.6
y	350	400	500	600

- (c) Define Gamma distribution. Given a Gamma random variable X with $r = 3$ and $\lambda = 2$. Compute $E(X)$, $Var(X)$ and $P(X \leq 1.5 \text{ years})$. 07

OR

- Q.5** (a) Define Skewness. The first four central moments of a distribution are 0, 2.5, 0.7 and 18.75. Compute Skewness and Kurtosis. 03

- (b) Two random samples gave the following data: 04

	Size	Mean	Variance
Sample I	8	9.6	1.2
Sample II	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population? $F_{0.05}(10,7) = 3.64$.

- (c) Fit a parabola $y = a + bx + cx^2$ for the following data: 07

x	1	2	3	4	5
y	5	12	26	60	97

Also, estimate y at x=6.