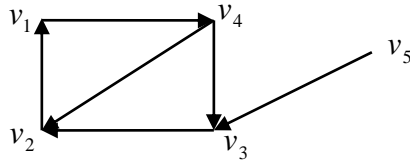


GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER- IV EXAMINATION – SUMMER 2020****Subject Code: 3140708****Date: 29/10/2020****Subject Name: Discrete Mathematics****Time: 10:30 AM TO 01:00 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

| | Marks |
|--|-----------|
| Q.1 (a) If $A = \{a, b\}$ and $B = \{c, d\}$ and $C = \{e, f\}$ then find (i) $(A \times B) \cup (B \times C)$ (ii) $A \times (B \cup C)$. | 03 |
| (b) Define even and odd functions. Determine whether the function $f : I \rightarrow R^+$ defined by $f(x) = 2x + 7$ is one-to-one or bijective. | 04 |
| (c) (i) Show that the relation $x \equiv y \pmod{m}$ defined on the set of integers I is an equivalence relation. | 03 |
| (ii) Draw the Hasse diagram for the partial ordering $\{(A, B) / A \subseteq B\}$ on the power set $P(S)$, where $S = \{a, b, c\}$. | 04 |
| Q.2 (a) Define equivalence class. Let R be the relation on the set of integers I defined by $(x - y)$ is an even integer, find the disjoint equivalence classes | 03 |
| (b) A committee of 5 persons is to be formed from 6 men and 4 women. In how many ways can this be done when (i) at least 2 women are included (ii) at most 2 women are included ? | 04 |
| (c) Solve the recurrence relation $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2$ using the method of undetermined coefficients. | 07 |
| OR | |
| (c) Solve the recurrence relation using the method of generating function $a_n - 5a_{n-1} + 6a_{n-2} = 3^n, n \geq 2; a_0 = 0, a_1 = 2$. | 07 |
| Q.3 (a) Define simple graph, degree of a vertex and complete graph. | 03 |
| (b) Define tree. Prove that there is one and only one path between every pair of vertices in a tree T . | 04 |
| (c) (i) A graph G has 15 edges, 3 vertices of degree 4 and other vertices of degree 3. Find the number of vertices in G . | 03 |
| (ii) Define vertex disjoint and edge disjoint subgraphs by drawing the relevant graphs. | 04 |
| OR | |
| Q.3 (a) Show that $(G, +_5)$ is a cyclic group, where $G = \{0, 1, 2, 3, 4\}$. | 03 |
| (b) Define the following by drawing graphs (i) weak component (ii) unilateral component (iii) strong component. | 04 |
| (c) (i) Construct the composite tables for (i) addition modulo 4 and (ii) multiplication modulo 4 for $Z_4 = \{0, 1, 2, 3\}$. Check whether they have identity and inverse element. | 03 |
| (ii) Define ring. Show that the set $M = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} / a, b \in R \right\}$ is not a ring under the operations of matrix addition and multiplication. | 04 |

- Q.4 (a)** Define algebraic structure, semi group and monoid. Also give related examples. **03**
- (b)** Use Warshall's algorithm to obtain path matrix from the adjacency matrix of **04**



- (c) (i)** Is the algebraic system $(Q, *)$ a group? Where Q is the set of rational numbers and $*$ is a binary operation defined by $a * b = a + b - ab, \forall a, b \in Q$. **03**
- (ii)** Let $(Z, +)$ be a group, where Z is the set of integers and $+$ is an operation of addition. Let H be a subgroup of Z consisting of elements multiple of 5. Find the left cosets of H in Z . **04**
- OR**
- Q.4 (a)** Prove that there are always an even number of vertices of odd degree in a graph. **03**
- (b)** Prove that every subgroup H of an abelian group is normal. **04**
- (c) (i)** Find the number of edges in a r -regular graph with n -vertices. **03**
- (ii)** A tree T has 4 vertices of degree 2, 4 vertices of degree 3, 2 vertices of degree 4. Find the number of pendant vertices in T . **04**
- Q.5 (a)** Show that the operation $*$ defined by $x * y = x^y$ on the set N of natural numbers is neither commutative nor associative. **03**
- (b)** Prove that an algebraic structure $(G, *)$ is an abelian group, where G is the set of non-zero real numbers and $*$ is a binary operation defined by $a * b = \frac{ab}{2}$. **04**
- (c) (i)** Find out using truth table, whether $(p \wedge r) \rightarrow p$ is a tautology. **03**
- (ii)** Obtain the dnf of the form $\neg(p \rightarrow (q \wedge r))$. **04**
- OR**
- Q.5 (a)** If $f : x \rightarrow 2x, g : x \rightarrow x^2$ and $h : x \rightarrow (x+1)$, then show that $(f \circ g) \circ h = f \circ (g \circ h)$. **03**
- (b)** Define lattice. Determine whether the POSET $(\{1,2,3,4,5\}; |)$ is lattice. **04**
- (c) (i)** If p : product is good, q : service is good, r : company is public limited, write the following in symbolic notations (i) either product is good or the service is good or both, (ii) either the product is good or service is good but not both, (iii) it is not the case that both product is good and company is public limited. **03**
- (ii)** For the universe of integers, let $p(x), q(x), r(x), s(x)$ and $t(x)$ be the following open statements: $p(x): x > 0$; $q(x): x$ is even; $r(x): x$ is a perfect square; $s(x): x$ is divisible by 4; $t(x): x$ is divisible by 5. Write the following statements in symbolic form: (i) At least one integer is even, (ii) There exists a positive integer that is even, (iii) If x is even then x is not divisible by 5, (iv) No even integer is divisible by 5, (v) There exists an even integer divisible by 5. **04**