Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III EXAMINATION – SUMMER 2020

Subject Code: 3130005 **Date:27/10/2020**

Subject Name: Complex Variables and Partial Differential Equations

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	If $u = x^3 - 3xy$ is find the corresponding analytic function $f(z) = u + iv$.	**03**
	(b)	Find the roots of the equation $z^2 - (5+i)z + 8 + i = 0$.	**04**
	(c)	(i) Determine and sketch the image of $\lvert z\rvert = 1$ under the transformation $w = z + i$.	**03**
		(ii) Find the real and imaginary parts of $f(z) = z^2 + 3z$.	**04**
Q.2	**(a)**	Evaluate $\int_C \left((x^2 - iy^2)dz \right)$ along the parabola $y = 2x^2$ from (1,2) to (2,8).	**03**
	(b)	Find the bilinear transformation that maps the points $z = \infty, i, 0$ into $w = 0, i, \infty$.	**04**
	(c)	(i) Evaluate $\oint_C \frac{e^{-z} dz}{z+1}$, where C is the circle $\lvert z\rvert = 1/2$.	**03**
		(ii) Find the radius of convergence of $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} z^n$.	**04**
		OR	
	(c)	(i) Find the fourth roots of -1.	**03**
		(ii) Find the roots of $\log z = i\frac{\pi}{2}$.	**04**
Q.3	**(a)**	Find $\oint_C \frac{1}{z^2} dz$, where $C : \lvert z\rvert = 1$.	**03**
	(b)	For $f(z) = \frac{1}{(z-1)^2 (z-3)}$, find Residue of *f(z)* at *z=1*.	**04**
	(c)	Expand $f(z) = \frac{1}{(z+2)(z+4)}$ in a Laurent series for the regions $(i) \lvert z\rvert < 2$, $(ii) 2 < \lvert z\rvert < 4$, $(iii) \lvert z\rvert > 4$.	**07**
		OR	
Q.3	**(a)**	Find $\oint_C \frac{z+4}{z^2 + 2z + 5} dz$, where $C : \lvert z+1\rvert = 1$.	**03**
	(b)	Evaluate using Cauchy residue theorem $\int_C \frac{e^{2z}}{(z+1)^3} dz$; C: $4x^2 + 9y^2 = 16$.	**04**
	(c)	Expand $f(z) = \frac{1}{z(z-1)(z-2)}$ in Laurent's series for the regions $(i) \lvert z\rvert < 1$, $(ii) 1 < \lvert z\rvert < 2$, $(iii) \lvert z\rvert > 2$.	**07**

Q.4 **(a)** Solve $xp + yq = x - y.$ **03**

(b) Derive partial differential equation by eliminating the arbitrary constants a and b from $z = ax + by + ab.$ **04**

(c) (i) Solve the p.d.e. $2r + 5s + 2t = 0.$ **03**

(ii) Find the complete integral of $p^2 = qz.$ **04**

OR

Q.4 **(a)** Find the solution of $x^2 p + y^2 q = z^2.$ **03**

(b) Form the partial differential equation by eliminating the arbitrary function ϕ from $z = \phi\left(\frac{y}{x}\right).$ **04**

(c) (i) Solve the p.d.e. $\left(D^2 - D'^2 + D - D'\right)z = 0.$ **03**

(ii) Solve by Charpit's method $yzp^2 - q = 0.$ **04**

Q.5 **(a)** Solve $(2D^2 - 5DD' + 2D'^2)z = 24(y - x).$ **03**

(b) Solve the p.d.e. $u_x + u_y = 2(x + y)u$ using the method of separation of variables. **04**

(c) Find the solution of the wave equation $u_{tt} = c^2 u_{xx}$, $0 \le x \le \pi$ with the initial and boundary conditions $u(0,t) = u(\pi,t) = 0; t > 0,$ $u(x,0) = k(\sin x - \sin 2x), u_t(x,0) = 0; 0 \le x \le \pi.$ $(c^2 = 1)$ **07**

OR

Q.5 **(a)** Solve the p.d.e. $r + s + q - z = 0.$ **03**

(b) Solve $2u_x = u_t + u$ given $u(x,0) = 4e^{-3x}$ using the method of separation of variables. **04**

(c) Find the solution of $u_t = c^2 u_{xx}$ together with the initial and boundary conditions $u(0,t) = u(2,t) = 0; t \ge 0$ and $u(x,0) = 10; 0 \le x \le 2.$ **07**