

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**BE - SEMESTER-III (NEW) EXAMINATION – WINTER 2021**

Subject Code: 3130005

Date: 17-02-2022

Subject Name: Complex Variables and Partial Differential Equations

Time: 10:30 AM TO 01:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

		Marks
Q.1	(a) Represent $z = 7i$ into polar form and find the argument of $z$ and the principal value of the argument of $z$ .	03
	(b) State De Moivre's theorem. Find and plot all roots of $(1+i)^{\frac{1}{3}}$ in the complex plane.	04
	(c) Using the method of separation of variables, solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ and $u = e^{-5y}$ when $x = 0$ .	07
Q.2	(a) Define an analytic function. Write the necessary and sufficient condition for function $f(z)$ to be analytic. Show that $f(z) =  z ^2$ is nowhere analytic.	03
	(b) Define the Mobious transformation. Determine the bilinear transformation which mapping the points $0, \infty, i$ onto $\infty, 2, 0$ .	04
	(c) <b>Attempt the following.</b>	
	(i) Define the harmonic function. Show that $u = x^2 - y^2 + x$ is harmonic and find harmonic conjugate of $u$ .	04
	(ii) Show that $f(z) = \begin{cases} \frac{\text{Im}(z)}{ z } & ; z \neq 0 \\ 0 & ; z = 0 \end{cases}$ is not continuous at $z = 0$	03
<b>OR</b>		
Q.3	(c) <b>Attempt the following.</b>	
	(i) Prove that $\cos^{-1} z = -i \ln(z + i\sqrt{1-z^2})$ .	4
	(ii) Find the values of $\text{Re } f(z)$ and $\text{Im } f(z)$ at the point $7+2i$ , where $f(z) = \frac{1}{1-z}$ .	3
Q.3	(a) Evaluate $\int_C \bar{z} dz$ , where $C$ is the right- half of the circle $ z  = 2$ and hence show that $\int_C \frac{dz}{z} = \pi i$ .	03
	(b) Expand $f(z) = \sin z$ in a Taylor series about $z = \frac{\pi}{4}$ and write the Maclaurin series for $e^{-z}$ .	04
	(c) Write the Cauchy integral theorem and Cauchy integral formula and hence evaluate:	07
	(i) $\oint_C \frac{e^z}{(z-1)(z-3)} dz; C:  z  = 2$ . (ii) $\oint_C e^z dz; C:  z  = 3$ .	

OR

- Q.3** (a) Evaluate  $\oint_C \frac{e^z}{z+i} dz$ , where  $C: |z-1|=1$ . 03
- (b) Develop the following functions into Maclaurin series: (i)  $\cos^2 z$  (ii)  $e^z \cos z$ . 04
- (c) Evaluate  $\int_C \operatorname{Re}(z^2) dz$ , where  $C$  is the boundary of the square with vertices  $0, i, 1+i, 1$  in the clockwise direction. 07

- Q.4** (a) Define the singular points of  $f(z)$ . Find the singularity of  $f(z)$  and classify as pole, essential singularity or removable singularity. where  $f(z) = \frac{1-e^z}{z}$ . 03
- (b) State the Cauchy residue theorem. Find the residue at its poles of  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  and hence evaluate  $\oint_C f(z) dz$ ,  $C: |z|=3$ . 04
- (c) Determine the Laurent series expansion of  $f(z) = \frac{1}{(z+2)(z+4)}$  valid for regions  
(i)  $|z| < 2$   
(ii)  $2 < |z| < 4$   
(iii)  $|z| > 4$  07

OR

- Q.4** (a) Solve  $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$ . 03
- (b) Form a partial differential equation by eliminating the arbitrary functions from the equations  $z = f(x+ay) + \phi(x-ay)$ . 04
- (c) Show that  $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$ . 07
- Q.5** (a) Form a partial differential equation by eliminating the arbitrary function from  $u = f\left(\frac{x}{y}\right)$ . 03
- (b) State the Lagrange's linear partial differential equation of first order and hence solve  $x(y-z)p + y(z-x)q = z(x-y)$  04
- (c) Using the method of separation of variables, solve  $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$  where  $u(0, y) = 8e^{-3y}$ . 07

OR

- Q.5** (a) Obtain the general solution of  $p + q^2 = 1$ . 03
- (b) Solve by Charpit's method:  $px + qy = pq$ . 04
- (c) Find the solution of the wave equation  $u_{tt} = c^2 u_{xx}$ ,  $0 \leq x \leq L$  satisfying the condition  $u(0, t) = u(L, t) = 0$ ,  $u_t(x, 0) = 0$ ,  $u(x, 0) = \frac{\pi x}{L}$ ,  $0 \leq x \leq L$ . 07

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