Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III(NEW) EXAMINATION – WINTER 2022

Subject Code: 3130005 **Date: 20-02-2023**

Subject Name: Complex Variables and Partial Differential Equations

Time: 02:30 PM TO 05:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	If $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$ then find its modulus value and the principal value of its argument.	**03**
	(b)	Find all the values of $(1-i)^{2/3}$	**04**
	(c)	Solve $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = \cos(x+2y)$	**07**
Q.2	**(a)**	Give examples of functions $f(z)$ and $g(z)$ which are analytic everywhere in the complex plane C such that $f(z)$ is never zero and $g(z) = 0$ if and only if $z = i$.	**03**
	(b)	Discuss the continuity of $$f(z) = \frac{Re(z^2)}{\lvert z\rvert^2}, \quad z \neq 0,$$ at $z = 0$ if $f(0) = 0$.	**04**
	(c)	Define Mobius transformation. Find the mobius transformation which maps the points $z = 0, -i, -1$ to $w = i, 1, 0$ respectively.	**07**
		OR	
	(c)	Define Harmonic function and show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic. State what are Cauchy Riemann equations and use it to find the harmonic conjugate of the given function $u(x,y)$.	**07**
Q.3	**(a)**	State: (i) Liouville Theorem and (ii) Cauchy-Goursat Theorem.	**03**
	(b)	Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$ using the appropriate formula.	**04**
	(c)	Evaluate $\int_C \bar{z}dz$ where C is along the sides of the triangle having vertices $z = 0, 1, i$.	**07**
		OR	
Q.3	**(a)**	Expand Ln $(1+z)$ as a Taylor series about $z = 0$ for $\lvert z\rvert < 1$.	**03**
	(b)	Without computing the integral, show that $$\left\lvert \int_C \frac{e^z}{z+1} dz \right\rvert \leq \frac{8\pi e^4}{3}$$ where C denotes the circle centered at origin and with radius 4.	**04**
	(c)	State the Generalized Cauchy integral formula and use it to compute $$\int_C \frac{z+e^z}{(z-1)^3} dz$$ where C is $\lvert z\rvert = 2$.	**07**

Q.4 **(a)** Identify the type of singularities of the function $f(z) = (z^2 - z^6)^{-1}$. **03**

(b) Using the Cauchy residue theorem, compute $\int_C \frac{1}{(z-1)^2(z-3)} dz$ where C is $|z| = 4$. **04**

(c) Solve $(x^2 + 2y^2)p - xyq = xz$. **07**

OR

Q.4 **(a)** Form the partial differentiation equation by eliminating the arbitrary constants from $(x-a)(x-b) - z^2 = x^2 + y^2$. **03**

(b) Find the complete integral (complete solution) of $q^2 = z^2p^2(1-p^2)$. **04**

(c) Find the Laurent's series of $f(z) = \frac{3}{(z-2)(z+1)}$ in the regions: (i) $|z| < 1$ and (ii) $1 < |z| < 2$. **07**

Q.5 **(a)** Solve $25\frac{\partial^2 z}{\partial x^2} - 40\frac{\partial^2 z}{\partial x \partial y} + 16\frac{\partial^2 z}{\partial y^2} = 0$. **03**

(b) Find the complete integral (complete solution) of $p^2 - q^2 = x - y$. **04**

(c) Solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x+y)u$ using the method of separation of variables. **07**

OR

Q.5 **(a)** Solve $\frac{\partial^2 z}{\partial x^2} + 10\frac{\partial^2 z}{\partial x \partial y} + 25\frac{\partial^2 z}{\partial y^2} = e^x e^{-y}$. **03**

(b) Using the contour integration, show that the value of the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ is equal to π. **04**

(c) A tightly stretched string with fixed end points $x = 0$ and $x = 1$ in the shape defined by $y = x(1-x)$ is released from this position of rest. Find $y(x,t)$ using the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **07**

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