Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III(NEW) EXAMINATION – SUMMER 2023

Subject Code:3130005 **Date:24-07-2023**

Subject Name: Complex Variables and Partial Differential Equations

Time: 02:30 PM TO 05:00 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Determine an analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$	**03**
	(b)	Solve the equation $z^2 + (2i-3)z + 5 - i = 0$	**04**
	(c)	Show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic and Find a harmonic conjugate of u(x,y)	**07**
Q.2	**(a)**	Evaluate $\int_C |z|^2 dz$ around the square with vertices at (0,0), (1,0), (1,1), (0,1).	**03**
	(b)	Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the following regions (i) $|z| < 2$ (ii) $2 < |z| < 4$	**04**
	(c)	(i) Evaluate $\int_C \frac{zdz}{(z-1)(z-2)}$ where C is the circle $|z| = \frac{1}{2}$	**03**
		(ii) Evaluate $\int_C \frac{dz}{z^2-7z+12}$ where C is the circle $|z| = 3.5$	**04**
		OR	
	(c)	Define mobius transformation. Determine the mobius transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ *onto* $w_1 = -1, w_2 = -i, w_3 = 1$	**07**
Q.3	**(a)**	Find and plot the image of triangular region in the z-plane with vertices (0,0), (1,0), (0,1) under the transformation $w = (1-i)z + 3$	**03**
	(b)	Find the values of a and b such that the function $f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$ is analytic.	**04**
	(c)	Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and Residue at each pole. Hence evaluate $\int_C f(z)dz$ where C is the circle $|z| = 3$	**07**
		OR	
Q.3	**(a)**	Expand $f(z) = \frac{1-e^z}{z}$ in Laurent's series about z = 0.	**03**
	(b)	Find modulus and argument of (i) $\frac{1+2i}{1-(1-i)^2}$ (ii) $\frac{(1+i)^2}{1-i}$	**04**
	(c)	Evaluate (i) $\int_C \frac{3z^2+7z+1}{z+1} dz$ Where C is $|z| = \frac{1}{2}$	**03**
		(ii) $\int_C \frac{z^2+1}{z^2-1} dz$ Where C is $|z-1| = 1$	**04**
Q.4	**(a)**	Solve $yzp - xzq = xy$	**03**
	(b)	Form partial differential equation by eliminating the arbitrary constants a and b from $z = axe^y + \frac{1}{2}a^2e^{2y} + b$	**04**
	(c)	(i) Solve $25r - 40s + 16t = 0$	**03**
		(ii) Solve $p^2 + q^2 = x + y$	**04**
		OR	
Q.4	**(a)**	Solve $(mz - ny)p + (nx - lz)q = ly - mx$	**03**

	(b)	Form a partial differential equation by eliminating the arbitrary functions from $f(x^2 - y^2, xyz) = 0$	**04**
	(c)	(i) Solve $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$	**03**
		(ii) Solve using Charpit's Method $z^2 = pqxy$	**04**
Q.5	**(a)**	Solve $(D^2 + 10DD' + 25D'^2)z = e^{3x+2y}$	**03**
	(b)	Solve $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$ using method of separation of variables.	**04**
	(c)	(i) Solve $(D^2 - D'^2)z = x - y$	**03**
		(ii) Solve $(2D^2 - 5DD' + 2D'^2) = \sin(2x + y)$	**04**

OR

Q.5	**(a)**	Solve $(1 - x)p + (z - y)q = 3 - z$	**03**
	(b)	Solve $2\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$ using method of separation of variables subject to the condition $u(x, 0) = 4e^{-3x}$	**04**
	(c)	Find the solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ such that $y = acospt \; when \; x = l \; and \; y = 0 \; when \; x = 0$	**07**

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