

Seat No : _____

Enrolment No: _____

GUJARAT TECHNOLOGICAL UNIVERSITY
DIPLOMA ENGINEERING – SEMESTER – I/II EXAMINATION – WINTER 2017

Subject Code: 3320002

Date:- 12-01-2018

Subject Name: Advanced Mathematics (Group-I)

Time: 10:30 AM to 01:00 PM

Total Marks:70

Instruction:

1. Attempt all question
2. Make Suitable assumption wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible.(Scientific/Higher Version not allowed)
5. English version is authentic.

Q.1 Fill in the blanks using appropriate choice from the given options.

યોગ્ય વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો.

[14]

1. $\sqrt{-9} = \underline{\hspace{2cm}}$.
(a) 3 (b) $\pm 3i$ (c) -3 (d) none of the above

૧. $\sqrt{-9} = \underline{\hspace{2cm}}$.
(અ) 3 (બ) $\pm 3i$ (ક) -3 (ડ) એક પણ નહીં

2. For $Z \in \mathbb{C}$ $Z - \bar{Z} = \underline{\hspace{2cm}}$.
(a) $\text{Im}(Z)$ (b) $i \text{Im}(Z)$ (c) $2\text{Im}(Z)$ (d) $2i \text{Im}(Z)$

૨. For $Z \in \mathbb{C}$ $Z - \bar{Z} = \underline{\hspace{2cm}}$.
(અ) $\text{Im}(Z)$ (બ) $i \text{Im}(Z)$ (ક) $2\text{Im}(Z)$ (ડ) $2i \text{Im}(Z)$

3. $(1+i)^2 = \underline{\hspace{2cm}}$.
(a) $2+2i$ (b) $2i$ (c) 2 (d) $2-2i$

૩. $(1+i)^2 = \underline{\hspace{2cm}}$.
(અ) $2+2i$ (બ) $2i$ (ક) 2 (ડ) $2-2i$

4. $Z = \frac{3}{5} - \frac{4}{5}i$ then $|z| = \underline{\hspace{2cm}}$.
(a) 1 (b) $-\frac{1}{5}$ (c) $\frac{7}{10}$ (d) $-\frac{7}{25}$

૪. $Z = \frac{3}{5} - \frac{4}{5}i$ then $|z| = \underline{\hspace{2cm}}$.
(અ) 1 (બ) $-\frac{1}{5}$ (ક) $\frac{7}{10}$ (ડ) $-\frac{7}{25}$

5. $\lim_{x \rightarrow 2} \frac{x^3+5}{5x+3} = \underline{\hspace{2cm}}$.

(a) $13/7$ (b) 1 (c) $3/4$ (d) $7/13$

4. $\lim_{x \rightarrow 2} \frac{x^3+5}{5x+3} = \underline{\hspace{2cm}}$

(a) $13/7$ (b) 1 (c) $3/4$ (d) $7/13$

6. $\lim_{x \rightarrow 0} \frac{\sin 4x}{x} = \underline{\hspace{2cm}}$.

(a) 0 (b) 1 (c) 4 (d) $1/4$

5. $\lim_{x \rightarrow 0} \frac{\sin 4x}{x} = \underline{\hspace{2cm}}$.

(a) 0 (b) 1 (c) 4 (d) $1/4$

7. $d/dx \sqrt{x} = \underline{\hspace{2cm}}$.

(a) $1/2\sqrt{x}$ (b) $2\sqrt{x}$ (c) $X^{\frac{3}{2}}$ (d) $X^{\frac{-3}{2}}$

9. $d/dx \sqrt{x} = \underline{\hspace{2cm}}$.

(a) $1/2\sqrt{x}$ (b) $2\sqrt{x}$ (c) $X^{\frac{3}{2}}$ (d) $X^{\frac{-3}{2}}$

8. $\frac{d}{dx}(x^3 + 3x + 5) = \underline{\hspace{2cm}}$.

(a) $3x^2 + 3$ (b) $\frac{x^4}{4} + \frac{3x^2}{2} + 5x$ (c) $3x+3$ (d) $3x^2 + 5$

6. $\frac{d}{dx}(x^3 + 3x + 5) = \underline{\hspace{2cm}}$.

(a) $3x^2 + 3$ (b) $\frac{x^4}{4} + \frac{3x^2}{2} + 5x$ (c) $3x+3$ (d) $3x^2 + 5$

9. $d/dx (\log \sqrt{x^2 + a^2}) = \underline{\hspace{2cm}}$.

(a) $\frac{x}{\sqrt{x^2+a^2}}$ (b) $\frac{2x}{\sqrt{x^2+a^2}}$ (c) $\frac{2x}{x^2+a^2}$ (d) $\frac{x}{x^2+a^2}$

6. $d/dx (\log \sqrt{x^2 + a^2}) = \underline{\hspace{2cm}}$.

(a) $\frac{x}{\sqrt{x^2+a^2}}$ (b) $\frac{2x}{\sqrt{x^2+a^2}}$ (c) $\frac{2x}{x^2+a^2}$ (d) $\frac{x}{x^2+a^2}$

10. $d/dx (x^x) = \underline{\hspace{2cm}}$.

(a) $x^x (1 + \log x)$ (b) $x + \log x$ (c) 1 (d) $x.x^{x-1}$

10. $d/dx (x^x) = \underline{\hspace{2cm}}$.

(અ) $x^x (1 + \log x)$ (બ) $x + \log x$ (ક) 1 (ડ) $x \cdot x^{x-1}$

11. $\int \sec 3x dx = \underline{\hspace{2cm}}$.

(a) $\frac{1}{3} \log (\sec 3x + \tan 3x)$ (b) $\log (\sec 3x + \tan 3x)$

(c) $\frac{1}{3} \log (\tan 3x)$ (d) $\frac{1}{3} \log (\sec 3x - \tan 3x)$

૧૧. $\int \sec 3x dx = \underline{\hspace{2cm}}$.

(અ) $\frac{1}{3} \log (\sec 3x + \tan 3x)$ (બ) $\log (\sec 3x + \tan 3x)$

(ક) $\frac{1}{3} \log (\tan 3x)$ (ડ) $\frac{1}{3} \log (\sec 3x - \tan 3x)$

12. $\int_2^5 x^2 dx = \underline{\hspace{2cm}}$.

(a) 39 (b) 117 (c) 125 (d) 133

૧૨. $\int_2^5 x^2 dx = \underline{\hspace{2cm}}$.

(અ) 39 (બ) 117 (ક) 125 (ડ) 133

13. order of difference equation $\frac{d^2y}{dx^2} + 4y = 0$ is $\underline{\hspace{2cm}}$.

(a) 1 (b) 2 (c) 0 (d) none of the above

૧૩. વિકલ સમીકરણ $\frac{d^2y}{dx^2} + 4y = 0$ ની કક્ષા $\underline{\hspace{2cm}}$ છે.

(અ) ૧ (બ) ૨ (ક) ૦ (ડ) એકપણ નહીં.

14. For differential equation $\frac{dy}{dx} + PY = Q$. I.F is $\underline{\hspace{2cm}}$.

(a) $e^{-\int p dx}$ (b) $e^{\int p dx}$ (c) $e^{\int Q dx}$ (d) none of the above

૧૪. વિકલ સમીકરણ $\frac{dy}{dx} + PY = Q$. સંકલ્પક અવયવ $\underline{\hspace{2cm}}$ છે.

(અ) $e^{-\int p dx}$ (બ) $e^{\int p dx}$ (ક) $e^{\int Q dx}$ (ડ) એકપણ નહીં.

Q 2 (A) Attempt any 2 out of 3. કોઈપણ બે ના જવાબ આપો

6

(1) Express into a + ib form $\frac{5+2i}{2+3i}$

(૧) $\frac{5+2i}{2+3i} a + ib$ સ્વરૂપમાં દર્શાવો

(2) Find square root of $3-4i$

(૨) $3-4i$ નું વર્ગમૂળ શોધો.

(3) Express into polar form $\frac{1+7i}{(2-i)^2}$

(૩). $\frac{1+7i}{(2-i)^2}$ ધ્રુવીય સ્વરૂપ માં દર્શાવો .

Q 2 (B) Attempt any 2 out of 3. કોઈપણ બે ના જવાબ આપો.

8

(1) If $f(x) = \frac{1+x}{1-x}$ prove that $f\left(\frac{x+y}{1+xy}\right) = f(x) \cdot f(y)$

(૧). જો $f(x) = \frac{1+x}{1-x}$ હોયતો સાબિત કરો $f\left(\frac{x+y}{1+xy}\right) = f(x) \cdot f(y)$.

(2) Evaluate: $\lim_{x \rightarrow a} \frac{\sqrt{2a-x}-\sqrt{x}}{a-x}$.

(૨) કીમત શોધો. $\lim_{x \rightarrow a} \frac{\sqrt{2a-x}-\sqrt{x}}{a-x}$.

(3) Evaluate $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3}$.

(૩). કીમત શોધો $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3}$

Q 3 (A) Attempt any 2 out of 3. કોઈપણ બે ના જવાબ આપો

6

(1) Differentiate $\sin x$ using definition.

(૧). વ્યાખ્યાની મદદથી $\sin x$ નું વિકલન શોધો.

(2) Differentiate $y = \frac{x^2 - 1}{x^2 + 1}$.

(૨). વિકલન શોધો $y = \frac{x^2 - 1}{x^2 + 1}$.

(3) Differentiate: $y = (\sin x)^x$.

(૩) વિકલન શોધો. $Y = (\sin x)^x$.

Q 3 (B) Attempt any 2 out of 3. કોઈ પણ બે ના જવાબ આપો

8

(1) Find $\frac{dy}{dx}$ for $y = x^3 \sin(\log x)$.

(૧) $y = x^3 \sin(\log x)$. હોય તો વિકલન $\frac{dy}{dx}$ શોધો.

(2) $y = \log(x + \sqrt{1+x^2})$ then prove that $(1+x^2)y_2 + xy_1 = 0$.

(૨) જો $y = \log(x + \sqrt{1+x^2})$ તો સાબિત કરો કે $(1+x^2)y_2 + xy_1 = 0$.

(3) For $f(x) = 2x^3 - 15x^2 + 36x + 10$ find maximum and minimum.

(૩) $f(x) = 2x^3 - 15x^2 + 36x + 10$ માટે અધિકતમ અને ન્યુનતમ કીમત શોધો.

Q 4 (A) Attempt any 2 out of 3. કોઈપણ બે ના જવાબ આપો

6.

(1) Integrate: $\int x e^x dx$.

(૧) સંકલન કરો. $\int x e^x dx$

(2) Integrate: $\int_0^1 \frac{2}{1+x^2} dx$

(૨) સંકલન કરો. $\int_0^1 \frac{2}{1+x^2}$

(3) Integrate: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$

(૩) સંકલન કરો. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$

Q 4 (B) Attempt any 2 out of 3. કોઈપણ બે ના જવાબ આપો

8

(1) Prove that area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab using integration.

(૧) નિયતસંકલનની મદદથી સાબિત કરો કે ઉપવલય $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ નું ક્ષેત્રફળ πab છે.

(2) Evaluate: $\int \frac{3x^2-2x}{x+4} dx .$

(૨) સંકલન કરો. $\int \frac{3x^2-2x}{x+4}$

(3) Evaluate: $\int_1^2 (x^2 + 4x + 1) dx$

(૩) સંકલન કરો. $\int_1^2 (x^2 + 4x + 1) dx$

Q 5 (A) Attempt any 2 out of 3. કોઈપણ બે ના જવાબ આપો.

6

(1.) Solve: $\frac{dy}{dx} + y \tan x = \sec x$

(૧.) ઉકેલો: $\frac{dy}{dx} + y \tan x = \sec x$

(2.) Solve: $(X^2 + 1) \frac{dY}{dx} + 2xy = 4x^2$

(૨.) ઉકેલો: $(X^2 + 1) \frac{dy}{dx} + 2xy = 4x^2$

(3.) Evaluate: $\lim_{X \rightarrow 0} \frac{5^x - 3^x}{x}$

(૩) કીમત શોધો. $\lim_{X \rightarrow 0} \frac{5^x - 3^x}{x}$

Q 5 (B) Attempt any 2 out of 3. કોઈપણ બે ના જવાબ આપો.

8

(1.) Solve: $\frac{dy}{dx} + x^2 \cdot e^{-y} = 0$

(૧.) ઉકેલો- $\frac{dy}{dx} + x^2 \cdot e^{-y} = 0$

(2.) Solve: $x \cos^2 y dx = y \cos^2 x dy$

(૨.) કીમત શોધો. $x \cos^2 y dx = y \cos^2 x dy$

(3.) Solve: $x \frac{dy}{dx} + 2y = \log x$

(૩) ઉકેલો- $x \frac{dy}{dx} + 2y = \log x .$