

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
DIPLOMA ENGINEERING – SEMESTER – 2(NEW) • EXAMINATION – SUMMER - 2018

**Subject Code: 3320002****Date: 23-May-2018****Subject Name: ADVANCED MATHEMATICS (GROUP-1)****Time: 10:30 AM TO 01:00 PM****Total Marks: 70****Instructions:**

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

**Q.1** Fill in the blanks using appropriate choice from the given options. **14**

1 If  $|\bar{z}| = 16$ , then  $|z| = \dots\dots\dots$   
 (a) 16 (b) 4 (c) 256 (d) 1

૧ જો  $|\bar{z}| = 16$  હોય, તો  $|z| = \dots\dots\dots$   
 (a) 16 (b) 4 (c) 256 (d) 1

2  $i^9 = \dots\dots\dots$   
 (a) i (b) -i (c) 1 (d) -1

૨  $i^9 = \dots\dots\dots$   
 (અ) i (બ) -i (ક) 1 (ડ) -1

3 If  $z = 5 - 2i$  then  $\bar{z} = \dots\dots\dots$   
 (a)  $-5 + 2i$  (b)  $-5 - 2i$  (c)  $5 + 2i$  (d) none of these

૩ જો  $z = 5 - 2i$  હોય તો  $\bar{z} = \dots\dots\dots$   
 (અ)  $-5 + 2i$  (બ)  $-5 - 2i$  (ક)  $5 + 2i$  (ડ) એક પણ નહીં

4 If  $f(x) = \log(\tan x)$  then  $f\left(\frac{\pi}{4}\right) = \dots\dots\dots$

૪ જો  $f(x) = \log(\tan x)$  તો  $f\left(\frac{\pi}{4}\right) = \dots\dots\dots$

(a) 1 (b) e (c) 0 (d)  $\pi$

5  $\lim_{\theta \rightarrow 0} \frac{\theta}{\tan 3\theta} = \dots\dots\dots$

(a) 3 (b)  $\frac{1}{3}$  (c) 1 (d) 0

૫  $\lim_{\theta \rightarrow 0} \frac{\theta}{\tan 3\theta} = \dots\dots\dots$

(a) 3 (b)  $\frac{1}{3}$  (c) 1 (d) 0

6  $\frac{d}{dx}(x^2+2^x+2^2) = \dots\dots\dots$   
 (a) 1 (b)  $2x+2^x+2^2$  (c)  $2x+2^x\log 2$  (d) 0

5  $\frac{d}{dx}(x^2+2^x+2^2) = \dots\dots\dots$   
 (a) 1 (b)  $2x+2^x+2^2$  (c)  $2x+2^x\log 2$  (d) 0

7 If  $y = e^x$ , then  $\frac{d^2y}{dx^2} = \dots\dots\dots$   
 (a)  $e^x$  (b)  $e^{2x}$  (c)  $e^{x^2}$  (d)  $e^{-x}$

9 જો  $y = e^x$  હોય, તો  $\frac{d^2y}{dx^2} = \dots\dots\dots$   
 (a)  $e^x$  (b)  $e^{2x}$  (c)  $e^{x^2}$  (d)  $e^{-x}$

8 If  $x = \cos \theta$ ,  $y = \sin \theta$  then  $\frac{dy}{dx} = \dots\dots\dots$   
 (a)  $\cot \theta$  (b)  $\tan \theta$  (c)  $-\cot \theta$  (d)  $-\tan \theta$

૮ જો  $x = \cos \theta$ ,  $y = \sin \theta$  તો  $\frac{dy}{dx} = \dots\dots\dots$   
 (અ)  $\cot \theta$  (બ)  $\tan \theta$  (ક)  $-\cot \theta$  (ડ)  $-\tan \theta$

9  $\frac{d}{dx} x^x = \dots\dots\dots$   
 (a)  $x - \log x$  (b)  $x + \log x$  (c)  $x^x(1 + \log x)$  (d)  $x \cdot x^{x-1}$

૯  $\frac{d}{dx} x^x = \dots\dots\dots$   
 (a)  $x - \log x$  (b)  $x + \log x$  (c)  $x^x(1 + \log x)$  (d)  $x \cdot x^{x-1}$

10  $\int \frac{1}{x^2} dx = \dots\dots\dots + c$   
 (a)  $\frac{1}{x}$  (b)  $\frac{-1}{x}$  (c)  $\frac{-1}{3x^3}$  (d)  $\frac{1}{3x^3}$

૧૦  $\int \frac{1}{x^2} dx = \dots\dots\dots + c$   
 (અ)  $\frac{1}{x}$  (બ)  $\frac{-1}{x}$  (ક)  $\frac{-1}{3x^3}$  (ડ)  $\frac{1}{3x^3}$

11  $\int \frac{1}{x^2 + 25} dx = \dots + c$

- (a)  $\tan^{-1}\left(\frac{x}{5}\right)$  (b)  $\frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right)$  (c)  $\frac{1}{5} \tan^{-1}\left(\frac{5}{x}\right)$  (d)  $\tan^{-1}\left(\frac{5}{x}\right)$

૧૧  $\int \frac{1}{x^2 + 25} dx = \dots + c$

- (અ)  $\tan^{-1}\left(\frac{x}{5}\right)$  (બ)  $\frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right)$  (ક)  $\frac{1}{5} \tan^{-1}\left(\frac{5}{x}\right)$  (ડ)  $\tan^{-1}\left(\frac{5}{x}\right)$

12  $\int_0^1 e^x dx = \dots$

- (a) e-1 (b) 1-e (c) e (d) -e

૧૨  $\int_0^1 e^x dx = \dots$

- (અ) e-1 (બ) 1-e (ક) e (ડ) -e

13 The order of a differential equation  $\frac{d^2y}{dx^2} = \left(3 + \frac{dy}{dx}\right)^3$  is .....

- (a) 3 (b) 2 (c) 1 (d) 6

૧૩ વિકલ્પ સમીકરણ  $\frac{d^2y}{dx^2} = \left(3 + \frac{dy}{dx}\right)^3$  ની કક્ષા ..... છે.

- (a) 3 (b) 2 (c) 1 (d) 6

14 The integrating factor(I.F.) of  $\frac{dy}{dx} + \frac{2y}{x} = e^x$  is .....

- (a)  $\frac{2}{x^2}$  (b)  $\frac{2}{x}$  (c)  $x^2$  (d)  $2x$

૧૪ વિકલ્પ સમીકરણ  $\frac{dy}{dx} + \frac{2y}{x} = e^x$  નો સંકલ્પકારક અવયવ (I.F.) ..... છે.

- (a)  $\frac{2}{x^2}$  (b)  $\frac{2}{x}$  (c)  $x^2$  (d)  $2x$

Q.2 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Find the modulus and principal argument of  $z = \sqrt{3} + i$  and express z into polar form.

૧.  $z = \sqrt{3} + i$  નો માનાંક અને કોણાંક શોધો, તથા z ને ધ્રુવિય સ્વરૂપમાં અભિવ્યક્ત કરો

2. Find the square root of  $3 - 4\sqrt{10}i$

૨.  $3 - 4\sqrt{10}i$  નું વર્ગમૂળ શોધો

3. Prove that  $(1 + \cos \theta + i \sin \theta)^n = 2^n \cos^n \left(\frac{\theta}{2}\right) \left[ \cos \frac{n\theta}{2} + i \sin \frac{n\theta}{2} \right]$

3. સાબિત કરો કે  $(1 + \cos \theta + i \sin \theta)^n = 2^n \cos^n \left( \frac{\theta}{2} \right) \left[ \cos \frac{n\theta}{2} + i \sin \frac{n\theta}{2} \right]$

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

1. If  $f(x) = \log x$ , then prove that

(i)  $f(x) + f(y) = f(xy)$  and (ii)  $f(x) - f(y) = f(x/y)$

૧. જો  $f(x) = \log x$  હોય, તો સાબિત કરો કે

(i)  $f(x) + f(y) = f(xy)$  and (ii)  $f(x) - f(y) = f(x/y)$

2. Evaluate :  $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3}$

૨.  $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3}$  મેળવો

3. Evaluate :  $\lim_{x \rightarrow 0} \frac{x \log(1+x)}{1 - \cos x}$

૩. ડક્ષ શોધો  $\lim_{x \rightarrow 0} \frac{x \log(1+x)}{1 - \cos x}$

Q.3 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Differentiate

$e^x$  with respect to  $x$  using first principle of differentiation

૧.  $e^x$  નું વિકલન  $x$  ની સાપેક્ષ વિકલનનાં પ્રથમ સિધ્ધાંત થી કરો

2. Find  $\frac{dy}{dx}$  if  $y = \log(\sec x + \tan x)$

૨. જો  $y = \log(\sec x + \tan x)$  તો  $\frac{dy}{dx}$  મેળવો

3. Equation of motion of a particle is  $s = t^3 - 6t^2 + 8t - 4$ . Then find the velocity and acceleration of the moving particle at  $t = 3$  second.

૩. કણની ગતિનું સમીકરણ  $s = t^3 - 6t^2 + 8t - 4$  છે. તો  $t = 3$  સેકન્ડે કણનો વેગ અને પ્રવેગ શોધો.

08

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

1. If  $x^3 + y^3 = 3axy$  then find  $\frac{dy}{dx}$

૧. જો  $x^3 + y^3 = 3axy$  તો  $\frac{dy}{dx}$  શોધો.

2. If  $x = \frac{1}{2}\left(t + \frac{1}{t}\right)$  and  $y = \frac{1}{2}\left(t - \frac{1}{t}\right)$  then find  $\frac{dy}{dx}$ .

૨. જો  $x = \frac{1}{2}\left(t + \frac{1}{t}\right)$  અને  $y = \frac{1}{2}\left(t - \frac{1}{t}\right)$  તો  $\frac{dy}{dx}$  મેળવો.

3. Find the maximum and minimum values of  $f(x) = 3x^3 - 4x^2 - x + 5$   
 ૩.  $f(x) = 3x^3 - 4x^2 - x + 5$  નાં મહત્તમ અને ન્યૂનતમ મૂલ્યો શોધો.

Q.4 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Find  $\int \cos x \cdot \sqrt{\sin x} dx$

૧. શોધો:  $\int \cos x \cdot \sqrt{\sin x} dx$

2. Find  $\int \frac{x}{(x+1)(x+2)} dx$

૨. શોધો :  $\int \frac{x}{(x+1)(x+2)} dx$

3. Evaluate:  $\int_0^{\frac{\pi}{2}} \log \cot x dx$

૩. કિંમત શોધો:  $\int_0^{\frac{\pi}{2}} \log \cot x dx$

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

1. Find  $\int x \cdot e^{3x} dx$

૧. શોધો :  $\int x \cdot e^{3x} dx$

2. Evaluate:  $\int_0^5 \frac{\sqrt[3]{x+2}}{\sqrt[3]{x+2} + \sqrt[3]{7-x}} dx$

૨. કિંમત શોધો:  $\int_0^5 \frac{\sqrt[3]{x+2}}{\sqrt[3]{x+2} + \sqrt[3]{7-x}} dx$

3. Find the area bounded by the curve  $y = x^2 - 7x + 10$  and x-axis .

૩. વક્ર  $y = x^2 - 7x + 10$  અને X-અક્ષ વચ્ચે ઘેરાયેલા પ્રદેશનું ક્ષેત્રફળ શોધો

Q.5 (a) Attempt any two કોઈપણ બે ના જવાબ આપો.

06

1. Evaluate :  $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

૧. મેળવો :  $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

2. Find the order and degree of  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho \left(\frac{d^2y}{dx^2}\right)^2$

૨.  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho \left(\frac{d^2y}{dx^2}\right)^2$  ની કક્ષા અને પરિમાણ મેળવો.

3. Form the differential equation whose general solution is  $Y = A \cos x + B \sin X$

૩. જેનો સામાન્ય ઉકેલ  $Y = A \cos x + B \sin X$  તે વિકલ સમીકરણ મેળવો.

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.

08

1. Solve :  $\frac{dy}{dx} = \frac{y}{x}$

૧. ઉકેલો :  $\frac{dy}{dx} = \frac{y}{x}$

2. Solve  $\frac{dy}{dx} = \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right)$ .

૨.  $\frac{dy}{dx} = \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right)$  ઉકેલો.

3. solve :  $x \log x \frac{dy}{dx} + y = \log x^2$

૩. ઉકેલો :  $x \log x \frac{dy}{dx} + y = \log x^2$

\*\*\*\*\*