

GUJARAT TECHNOLOGICAL UNIVERSITY
DIPLOMA ENGINEERING – SEMESTER-II EXAMINATION – Summer- 2019

Subject Code: 3320002

Date: 03-06-2019

Subject Name: ADVANCED MATHEMATICS (GROUP-1)

Time: 10:30 AM to 01:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of programmable & Communication aids are strictly prohibited.
5. Use of only simple calculator is permitted in Mathematics.
6. English version is authentic.

Q.1

Fill in the blanks using appropriate choice from the given options.

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(યોગ્ય વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો.)

- 1 If $3x + 2yi = 6 + 4i$ then $x = \underline{\hspace{2cm}}$, $y = \underline{\hspace{2cm}}$
 (જો $3x + 2yi = 6 + 4i$ હોય તો $x = \underline{\hspace{2cm}}$, $y = \underline{\hspace{2cm}}$)
 (a) 2, 2 (b) 2, 3 (c) 3, 2 (d) 1, 1
- 2 $i + i^2 + i^3 + i^4 = \underline{\hspace{2cm}}$
 (a) -1 (b) 0 (c) 1 (d) i
- 3 If $z = (3 - 4i)$ then $|z| = \underline{\hspace{2cm}}$
 (જો $z = (3 - 4i)$ હોય તો $|z| = \underline{\hspace{2cm}}$)
 (a) -5 (b) 25 (c) 5 (d) 0
- 4 If $f(x) = x^3 + 5$ then $f(2) = \underline{\hspace{2cm}}$
 (a) 13 (b) 5 (c) 0 (d) -3
- 5 If $f(x) = \log x$ then $f(x) + f(y) = \underline{\hspace{2cm}}$
 (જો $f(x) = \log x$ હોય તો $f(x) + f(y) = \underline{\hspace{2cm}}$)
 (a) $f(x + y)$ (b) $f(x - y)$ (c) $f(x \cdot y)$ (d) $f\left(\frac{x}{y}\right)$
- 6 $\lim_{x \rightarrow 1} (x^3 - 3x^2 + 5x - 6) = \underline{\hspace{2cm}}$
 (a) 1 (b) 6 (c) 3 (d) -3
- 7 $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} = \underline{\hspace{2cm}}$
 (a) -12 (b) 0 (c) 8 (d) 12
- 8 $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = \underline{\hspace{2cm}}$
 (a) -1 (b) 0 (c) 1 (d) ∞
- 9 $\frac{d}{dx} (\sec x) = \underline{\hspace{2cm}}$
 (a) $\sec^2 x$ (b) $-\sec^2 x$ (c) $\sec x \cdot \tan x$ (d) $-\sec x \cdot \tan x$
- 10 $\frac{d}{dx} (\sin^2 x + \cos^2 x) = \underline{\hspace{2cm}}$
 (a) 1 (b) -1 (c) 0 (d) none of these
- 11 $\int x^7 dx = \underline{\hspace{2cm}} + c$
 (a) $\frac{x^8}{8}$ (b) $\frac{x^6}{6}$ (c) $7x^6$ (d) $7 \log x$
- 12 $\int_{-2}^2 x^3 dx = \underline{\hspace{2cm}}$
 (a) -1 (b) 0 (c) 1 (d) $\frac{1}{4}$

- 13 The order of the differential equation $\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^3 + 4y = 0$ is _____
(વિકલ સમીકરણ $\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^3 + 4y = 0$ ની કક્ષા _____ થશે.)
(a) 1 (b) 0 (c) 2 (d) 3
- 14 The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 5\left(\frac{dy}{dx}\right)^2 - y = 0$ is _____
(વિકલ સમીકરણ $\left(\frac{d^2y}{dx^2}\right)^3 + 5\left(\frac{dy}{dx}\right)^2 - y = 0$ નું પરિમાણ _____ થશે.)
(a) 1 (b) 3 (c) 0 (d) 2

Q.2 (a) Attempt any two. (કોઈપણ બે ગણો.) 06

1. Simplify (સાદુરૂપ આપો):

$$\frac{(\cos 3\theta + i \sin 3\theta)^{-4} (\cos \theta - i \sin \theta)^5}{(\cos 2\theta - i \sin 2\theta)^6 (\cos 13\theta + i \sin 13\theta)}$$

2. If $z = \frac{3+7i}{1-i}$ then find its conjugate complex number and modulus.
(જો $z = \frac{3+7i}{1-i}$ હોય તો તેની અનુબદ્ધ સંકર સંખ્યા અને માનક શોધો)

3. Find the value (કિંમત શોધો): $\lim_{x \rightarrow 0} \frac{4^x - 3^x}{x}$

(b) Attempt any two. (કોઈપણ બે ગણો.) 08

1. Find square root of $3 + 4\sqrt{10}i$

($3 + 4\sqrt{10}i$ નું વર્ગમૂળ શોધો.)

2. Convert $1 - \sqrt{3}i$ into polar form.

($1 - \sqrt{3}i$ ને ધ્રુવીય સ્વરૂપમાં ફેરવો.)

3. If $f(x) = \log\left(\frac{x-1}{x}\right)$ then prove that $f(x) + f(-x) = f(x^2)$.

(જો $f(x) = \log\left(\frac{x-1}{x}\right)$ હોય તો સાબિત કરો કે $f(x) + f(-x) = f(x^2)$.)

Q.3 (a) Attempt any two. (કોઈપણ બે ગણો.) 06

1. $\lim_{x \rightarrow -1} \frac{2x^3 + 5x^2 + 4x + 1}{3x^3 + 5x^2 + x - 1}$

2. $\lim_{n \rightarrow \infty} \frac{3n^3 - 4n^2 - n - 5}{2n^3 + 3n^2 - 2n + 7}$

3. (i) $\lim_{\theta \rightarrow 0} \frac{\sin 2\theta}{\tan 7\theta}$ (ii) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{2x}$

(b) Attempt any two. (કોઈપણ બે ગણો.) 08

1. Solve the differential equation (વિકલ સમીકરણ ઉકેલો)

$$x(1+y^2)dx = y(1+x^2)dy$$

2. Solve the differential equation (વિકલ સમીકરણ ઉકેલો)

$$\tan y dx + \tan x \cdot \sec^2 y dy = 0$$

3. Solve the differential equation (વિકલ સમીકરણ ઉકેલો)

$$\frac{dy}{dx} + 2y = e^x$$

Q.4 (a) Attempt any two. (કોઈપણ બે ગણો.) 06

1. If $x = e^t + \sin t, y = \log t + \cos t$ then find $\frac{dy}{dx}$.

(જો $x = e^t + \sin t, y = \log t + \cos t$ હોય તો $\frac{dy}{dx}$ શોધો.)

2. Find velocity (v) and acceleration (a) at $t = 2$ for the equation of motion

$$s = 2t^3 + 3t^2 - 12t + 5.$$

(ગતિસૂત્ર $s = 2t^3 + 3t^2 - 12t + 5$ હોય તો વેગ (v) અને પ્રવેગ (a), $t = 2$ આગળ શોધો.)

3. If $y = A\cos pt + B\sin pt$ then prove that $y'' + p^2y = 0$.
(જો $y = A\cos pt + B\sin pt$ હોય તો સાબિત કરો કે $y'' + p^2y = 0$.)

(b) Attempt any two. (કોઈપણ બે ગણો.)

08

1. Differentiate $y = e^x$ using the definition.

($y = e^x$ નું વ્યાખ્યાની મદદથી વિકલન શોધો.)

2. If $y = (\sin x)^x$ then find y' . (જો $y = (\sin x)^x$ હોય તો y' શોધો.)

3. Find the maximum and minimum value of $f(x) = 2x^3 - 15x^2 + 36x + 10$.

($f(x) = 2x^3 - 15x^2 + 36x + 10$ માટે મહત્તમ અને ન્યૂનતમ કિંમત શોધો.)

Q.5

(a) Attempt any two. (કોઈપણ બે ગણો.)

06

1. $\int \frac{x-5}{(x-1)(x-2)} dx$

2. $\int_1^1 \frac{x^3 - 27}{x - 3} dx$

3. $\int_{-1}^{-1} \cos 5x \cdot \sin 2x dx$

(b) Attempt any two. (કોઈપણ બે ગણો.)

08

1. $\int_0^1 \frac{dx}{x + \sqrt{1-x^2}}$

2. $\int e^x x^3 dx$

3. Find the area bounded by the curve $ay = x^2$, line $x = a$ and X-axis.

(વક્રો $ay = x^2$, રેખા $x = a$ અને X-અક્ષ વચ્ચે ઘેરાયેલા પ્રદેશનું ક્ષેત્રફળ શોધો.)