Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA - SEMESTER– III EXAMINATION – WINTER 2019

Subject Code: 4639301 **Date: 17/12/2019**

Subject Name: Basic Mathematics

Time: 10:30 AM TO 01:00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Give definition of the following terms: **07**

i) Union of two sets
ii) Symmetric Matrix
iii) Complete Graph
iv) Modus Tollens
v) Transitive Property of a Relation
vi) Domain of a function
vii) Binary Tree

(b) Define Tautology. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology without using the Truth Table. **07**

Q.2 **(a)** (I) Verify the validity of the following arguments: **04**
"All men are mortal. Socrates is a man. Therefore, Socrates is mortal".

(II) Let $A = \{0,2,4,6,8,10\}$, $B = \{0,1,2,3,4,5,6\}$ and $C = \{4,5,6,7,8,9,10\}$. Find (i) $(A \cup B) \cap C$ (ii) $(A \cup B) \cup (A \cap C)$ (iii) $(A \cap B) \cup (A \cap C)$ **03**

(b) Using truth table and logical equivalences show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent. **07**

OR

(b) Define Relation on a set. For each of these relations on the set $\{1,2,3,4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive. **07**

(i) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$ (ii) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$
(iii) $\{(2,4),(4,2)\}$ (iv) $\{(1,2),(2,3),(3,4)\}$ (v) $\{(1,1),(2,2),(3,3),(4,4)\}$
(vi) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

Q.3 **(a)** Give a direct proof of the theorem "If n is an odd integer, then n^2 is odd." **07**

(b) Define the transpose of a Matrix. Consider the given matrix as A. Find the product of the matrix. i.e. A^2 **07**

$$\begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{bmatrix}$$

OR

Q.3 **(a)** Use the Principal of Mathematical Induction to prove : **07**

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for the positive integer n.

(b) Define gcd and lcm of integers. What is the gcd and lcm of 92928 and 123552. **07**

Q.4 **(a)** Define one-one, Onto function. Let f and g be the functions from the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. What is the composition of function f and g? What is the composition of g and f? **07**

(b) **07**

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

Find (i) $A \vee B$ (ii) $A \wedge B$ (iii) $A \odot B$

OR

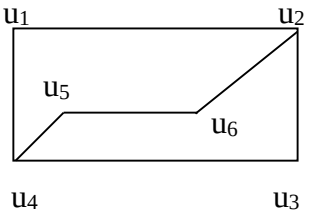
Q.4 **(a)** Define directed graph. Draw the directed graph for each of the following relations on the set $\{1, 2, 3, 4\}$: **07**

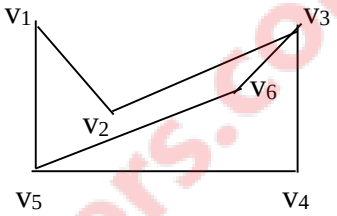
(i) $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$ (ii) $\{(1,1),(1,4),(2,2),(3,3),(4,1)\}$

(iii) $\{(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,1),(4,2),(4,3)\}$

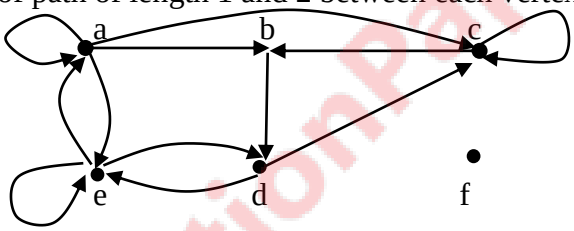
(iv) $\{(2,4),(3,1),(3,2),(3,4)\}$

(b) Define graph isomorphism. Determine whether the graphs G and H shown below are isomorphic. **07**

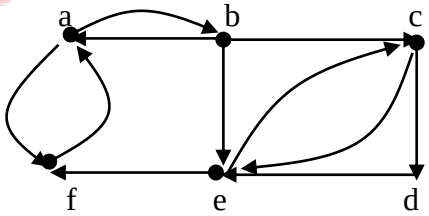




Q.5 **(a)** For the given graph below find in-degree and out-degree of each vertex and also find the given adjacency matrix. Using this adjacency matrix, find total no. of path of length 1 and 2 between each vertex. **07**



(b) Define Strong, Unilateral and Weak component. Also find Strong, Unilateral and Weak component from the given digraph **07**



OR

Q.5 **(a)** Define Binary Tree. Draw the subgraph of tree represented by: **07**
(A(B(C(D)(E))(F(G)(H(J))))(K(L)(M(N)(P(Q)(R)))))
Obtain a binary tree corresponding to it.

(b) Write the order of nodes for the tree mentioned in above example (OR:Q5(a)) if it is traversed in **07**
(i) preorder (ii) inorder (iii) postorder

****