

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**ME – SEMESTER – III (New) • EXAMINATION – WINTER - 2020**

**Subject Code: 3730007****Date: 1/1/2021****Subject Name: Operation Research****Time: 10:30 AM TO 12:30 PM****Total Marks: 56****Instructions:**

1. Attempt any FOUR questions out of EIGHT questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

**Q.1 (a) (i)** What is unimodal function? How unimodality is used in elimination techniques? **Marks 03**

**(ii)** Write a dual to the following LP problem. **04**

$$\text{Minimize } Z = x_1 + 2x_2$$

Subject to the constraints

$$(i) 2x_1 + 4x_2 \leq 160, (ii) x_1 - x_2 = 30, (iii) x_1 \geq 10$$

$$\text{and } x_1, x_2 \geq 0$$

**(b)** An established company has decided to add a new product to its line. **07**  
 It will buy the product from a manufacturing concern, package it, and sell it to a few distributors that have been selected on a geographical basis. Market research has already indicated the volume expected and the size of sales force required. The steps shown in the following table are to be planned.

Activity	Description	Predecessors	Duration (days)
A	Organize sales office	--	6
B	Hire salesmen	A	4
C	Train salesmen	B	7
D	Select advertising agency	A	2
E	Plan advertising campaign	D	4
F	Conduct advertising campaign	E	10
G	Design package	--	2
H	Setup packaging facilities	G	10
I	Package initial stocks	J, H	6
J	Order stock from manufacturer	--	13
K	Select distributors	A	9
L	Sell to distributors	C, K	3
M	Ship stocks to distributors	I, L	5

(a) Draw a network diagram for this project

(b) Indicate the critical path

(c) For each non-critical activity, find the total and free float.

**Q.2 (a) (i)** What is dynamic programming? Explain state and stage. **03**

**(ii)** In a game of matching coins with two players, suppose A wins one unit of value when there are two heads, wins nothing when there are two tails and losses  $\frac{1}{2}$  unit of value when there is one head and one tail. Determine the payoff matrix, the best strategies for each player and the value of the game to A by using algebraic method. **04**

- (b) Solve the following problem by using the method of Lagrangian multipliers. 07

$$\text{Minimize } Z = x_1^2 + x_2^2 + x_3^2$$

Subject to the constraints

$$(i) x_1 + x_2 + 3x_3 = 2, (ii) 5x_1 + 2x_2 + x_3 = 5$$

and

$$x_1, x_2 \geq 0$$

- Q.3** (a) Use penalty (Big-M) method to solve the following LP problem 07

$$\text{Minimize } Z = 5x_1 + 3x_2$$

Subject to the constraints

$$(i) 2x_1 + 4x_2 \leq 12, (ii) 2x_1 + 2x_2 = 10, (iii) 5x_1 + 2x_2 \geq 10$$

$$\text{and } x_1, x_2 \geq 0$$

- (b) A company that operates for 50 weeks in a year is concerned about its stocks of copper cable. This costs Rs 240 a meter and there is a demand for 8000 meters a week. Each replenishment costs Rs 1050 for administration and Rs 1650 for delivery, while holding costs are estimated at 25 per cent of value held a year. Assuming no shortage are allowed, what is the optimal inventory policy for the company? How would this analysis differ if the company wanted to maximize its profits rather than minimize cost? What is the gross profit if the company sells the cable for Rs 360 a meter? 07

- Q.4** (a) Use simplex method to solve the following LP problem: 07

$$\text{Maximize } Z = 3x_1 + 5x_2$$

Subject to the constraints

$$(i) 3x_1 + 2x_2 \leq 18, (ii) x_1 \leq 4, (iii) x_2 \leq 6$$

$$\text{and } x_1, x_2 \geq 0$$

Discuss the change in  $c_j$  on the optimality of the optimal basic feasible solution.

- (b) A glass factory that specializes in crystal is developing a substantial backlog and for this the firm's management is considering three courses of action: To arrange for subcontracting ( $S_1$ ), to begin overtime production ( $S_2$ ), and to construct new facilities ( $S_3$ ). The correct choice depends largely upon the future demand, which may be low, medium, or high. By consensus, management ranks the respective probabilities as 0.10, 0.50 and 0.40. A cost analysis reveals the effect upon the profits. This is shown in the table below: 07

Demand	Probability	Courses of Action		
		$S_1$	$S_2$	$S_3$
Low (L)	0.10	10	-20	-150
Medium (M)	0.50	50	60	20
High (H)	0.40	50	100	200

Show this decision situation in the form of a decision tree and indicate the most preferred decision and its corresponding expected value.

- Q.5** (a) Write a note on crashing of the network. 07

- (b) Solve the following LP problem using Graphical method. 07

$$\text{Maximize } Z = 7x_1 + 3x_2$$

Subject to the constraints

$$(i) x_1 + 2x_2 \geq 3, (ii) x_1 + x_2 \leq 4, (iii) 0 \leq x_1 \leq \frac{5}{2}, (iv) 0 \leq x_2 \leq \frac{3}{2}$$

$$\text{and } x_1, x_2 \geq 0$$

- Q.6 (a)** The demand for an item in a company is 18,000 units per year, and the company can produce the item at a rate of 3,000 per month. The cost of one set-up is Rs 500 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs 240 per year. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and the time between set-ups. **07**

- (b)** Solve the following Integer LP problem using Gomory's cutting plane method. **07**

*Maximize*  $Z = x_1 + x_2$

Subject to the constraints

(i)  $3x_1 + 2x_2 \leq 5$ , (ii)  $x_2 \leq 2$

and  $x_1, x_2 \geq 0$  and are integers.

Solution to the LP problem by the simplex method is given as below:

		$c_j \rightarrow$	1	1	0	0
Basic variables coefficient $C_B$	Basic variables B	Basic variables value b ( $=x_B$ )	$x_1$	$x_2$	$s_1$	$s_2$
1	$x_1$	1/3	1	0	1/3	-2/3
1	$x_2$	2	0	1	0	1
$Z=7/2$		$c_j - z_j$	0	0	-1/3	-1/3

- Q.7 (a)** Find a minimum of  $f = x(x - 1.5)$  in the interval (0.0, 1.00), considering  $\delta = 0.001$  and  $n = 6$  as the number of experiments by using Dichotomous Search Method. **07**

- (b)** Minimize  $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$  with the starting point (0, 0), considering  $\epsilon = 0.01$ , up to two iteration, taking search direction  $s_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $s_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ , using Univariate Method. **07**

- Q.8 (a)** *Minimize*  $f(x) = 0.65 - \left[ \frac{0.75}{(1+x^2)} \right] - 0.65x \tan^{-1} \left( \frac{1}{x} \right)$  in the interval [0, 3] by the Fibonacci method using  $n = 6$ . Also, find ratio of the final to the initial interval of uncertainty. **07**

- (b)** What is sequential linear programming? Explain its advantages. **07**