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# GUJARAT TECHNOLOGICAL UNIVERSITY ME - SEMESTER-1 (NEW) EXAMINATION - WINTER 2018 

## Subject Code: 3710214 <br> Date: 01/01/2019 <br> Subject Name: Mathematical foundations of Computer Science <br> Time: 02:30 PM To 05:00 PM <br> Total Marks: 70

## Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full mark.
Q. 1 (a) A man draws 3 balls from an urn containing 5 white and 7 black balls. He gets Rs. 10 for each white ball and Rs. 5 for each black ball. Find his expectation.
(b) Define Isomorphic graph. Are the two graphs given in the figure, is isomorphic?

(c) In a manufacturing company, a one-month training program is conducted to each employee. A new training program has been developed. To test the new program, 2 groups of 9 employees each were assigned a job, the first group trained under the old system and the second trained under the new program. Their performance timings in minutes are recorded as follows:

| X (old <br> program) | 31 | 38 | 36 | 30 | 42 | 34 | 32 | 33 | 39 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y (New <br> program) | 34 | 33 | 26 | 36 | 31 | 28 | 27 | 29 | 35 |

Do the job execution times present sufficient evidence to indicate that the mean time is less for the new training program? [ $\mathrm{t}_{0.05,16}=$ 1.746]
Q. 2 (a) In a large city A, 20\% of a random sample of 900 school children had defective eye sight. In another large city B, $15 \%$ of a random sample of 1600 children had the same defect. Obtain $95 \%$ confidence limits for the difference in the population proportions.
(b) What is machine learning? write a short note. Explain commonly used performance measures for machine learning algorithm?
(c) Four losses are observed from a Gamma distribution. The observed losses are 200, 300, 350 and 450 . Find a method of moments estimate for $\alpha$.

## OR

(c) Suppose that X is discrete random variable with the following PMF

| X | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{X})$ | $2 \theta / 3$ | $\theta / 3$ | $2(1-\theta) / 3$ | $(1-\theta) / 3$ |

Where $0 \leq \theta \leq 1$ is a parameter. The following 10 independent observations $3,0,2,1,3,2,1,0,2,1$ were taken from such a distribution. What is the maximum likelihood estimate of $\theta$.
Q. 3 (a) Given the PDF of a continuous random variable x as follows

$$
f(x)= \begin{cases}6 x(1-x), & 0<x<1 \\ 0, & \text { otherwise }\end{cases}
$$

Find Cumulative Density Function (CDF) for x .
(b) Explain principal component analysis in brief.
(c) Define Hamiltonian path, Hamiltonian cycle and Hamiltonian graph. Show that given graph G has no Hamiltonian cycle but the graph has a Hamiltonian path and draw it.


## OR

Q. 3 (a) The joint PDF of (X,Y) is given by

$$
f(x, y)=\left\{\begin{array}{rc}
24 x y, & x>0, y>0, x+y \leq 1 \\
0, & \text { otherwise }
\end{array}\right.
$$

Find the conditional mean and variance of Y given X
(b) What is classification problem? List different classification methods and explain any one in detail.
(c) Define planar graph and graph coloring. Show that the given graph is planar. Also, prove that $\mathrm{K}_{5}$ is non-planar graph.

Q. 4 (a) The mean and variance of a binomial distribution are 4 and 3 respectively. Find $p(x=0), p(x=1)$ and $p(x \geq 2)$.
(b) If X is a random variable of the number of heads obtained in tossing three coins, Prove that,

$$
p\left(\left|X-\frac{3}{2}\right| \geq 2\right) \leq \frac{3}{16}
$$

(c) What is data mining? Explain the role of probability in data mining.

Also, explain what is support and confidence in data mining?

## OR

Q. 4 (a) Wireless sets are manufactured with 25 soldered joints each on the average 1 defective joint in 500 . How many sets can be expected to be free from defective joints in a consignment of 10,000 sets?
(b) Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Color the line segment joining every pair of adjacent points by blue and the rest by red. If the numbers of red and blue line segment are equal, then what is the value of $n$ ?
(c) How mathematical concepts are being used to implement security algorithms? Explain in detail.
Q. 5 (a) An electric firm manufactures light bulbs that have life before burning out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. $[\mathrm{p}(0<Z<0.55)=0.2088$ and $\mathrm{p}(0<\mathrm{Z}<0.85)=0.3023]$
(b) Explain application of mathematics in the field of software engineering.
(c) What is bioinformatics? What is the role of probability distribution in bioinformatics?

## OR

Q. 5 (a) A random Sample of size 100 is taken from a population whose mean is 60 and variance is 400 . Using Central Limit Theorem, find with what probability can we assert that the mean of the sample will not differ from $\mu=60$ by more than 4 . $[p(0 \leq z \leq 2)=0.4773$ ]
(b) List and explain different network protocols based on mathematical foundation.
(c) Write a short note on mathematical tools of Soft Computing.

