

**GUJARAT TECHNOLOGICAL UNIVERSITY****ME – SEMESTER – I (New)– EXAMINATION – WINTER-2019****Subject Code: 3710214****Date: 02-01-2020****Subject Name: Mathematical foundations of Computer Science****Time: 02:30 PM TO 05:00 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** If the probability that an individual suffer a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals: **07**

1) Exactly 3    2) more than 2 individuals    3) None    4) More than 1 individual will suffer a bad-reaction.

- (b)** What is the significance of Method of moments in statistics? Consider Following sample data follows binomial distribution  $[b(x; n, P) = n * P^x * (1 - P)^{n-x}]$ . Estimate the value of n and p using method of moments? **07**

2, 3, 2, 0, 1, 3, 3, 3, 3, 1, 3, 3, 3, 2, 1

- Q.2 (a)** The joint distribution of X and Y is given in table. Find  $E(Y|X=1)$ . **07**

	X=0	X=1	X=3
Y= -1	0.11	0.03	0.00
Y=2.5	0.03	0.09	0.16
Y=3	0.15	0.15	0.06
Y=4.7	0.04	0.16	0.02

- (b)** The assets (in billions of dollars) of the four wealthiest people in a particular country are 44, 34, 33, and 14. Assume that samples of size 2 are randomly selected with replacement from this population of four values. **07**

1) After listing the possible samples and finding the mean of each sample, use a table to describe the sampling distribution of the sample means.

2) Find the mean of the sampling distribution.

3) Do the sample means target the value of the population mean? In general, do sample means make good estimators of population mean? Why or why not?

**OR**

- (b)** A random variable X has the following probability function : **07**

x	0	1	2	3	4
P(x)	k	3k	5k	7k	9k

Find : 1) The value of k    2)  $P(X < 3)$  and  $P(0 < X < 4)$     3) The distribution function of X

- Q.3 (a)** What is the significance of building a model? Explain the problem of model over-fitting. How to avoid over-fitting problem? **07**

- (b)** Use PCA to reduce the dimensions **07**

X	2	1	0	-1
Y	4	3	1	0.5

**OR**

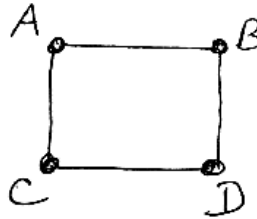
- Q.3 (a)** If a person of age 48 years take a loan amount of Rs 142000, classify the probability of person falling in the defaulter class based on the previous experience **07**

Age	25	35	45	20	35	52	23	40	60	48	33
Loan (thousand)	40	60	80	20	120	18	95	62	100	220	150
Defaulter	N	N	N	N	N	N	Y	Y	Y	Y	Y

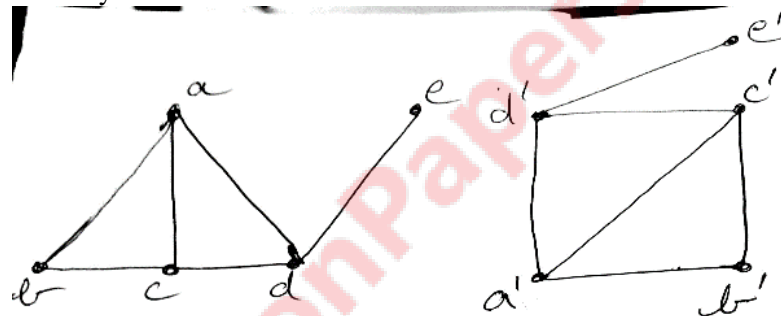
- (b) Analyze the following 3-variate data-set with 10 observations. Each observation consists of 3 measurements on a wafer: thickness, horizontal displacement and vertical displacement. Use PCA to reduce the dimensions 07

$$\mathbf{X} = \begin{array}{|c|c|c|} \hline 7 & 4 & 3 \\ \hline 4 & 1 & 8 \\ \hline 6 & 3 & 5 \\ \hline 8 & 6 & 1 \\ \hline 8 & 5 & 7 \\ \hline 7 & 2 & 9 \\ \hline 5 & 3 & 3 \\ \hline 9 & 5 & 8 \\ \hline 7 & 4 & 5 \\ \hline 8 & 2 & 2 \\ \hline \end{array}$$

- Q.4 (a) How many ways the below graph can be coloured if almost 4 colours can be used? 07

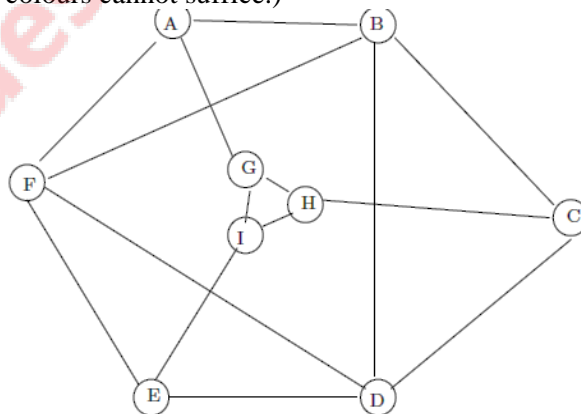


- (b) Are the two graphs shown below isomorphic? If so, give the isomorphism; if not, give careful reasons for your answer. 07



OR

- Q.4 (a) Compute the chromatic number (vertex colouring number) of the graph G shown below. Justify your answer! (Show a colouring with (G) colours (label each node with its colour), and argue that fewer colours cannot suffice.) 07



- (b) Solve the following crime problem with graph theory: 07  
 4 people named as A B C and D are being investigated. Only one person is telling the truth. From the statements of these 4 find out who is the thief.  
 1) A says "I 'm not the thief"  
 2) B says "A is the thief"  
 3) C says "I 'm not the thief"  
 4) D says "B is the thief".

**Q.5 (a)** Find the value of  $c$  and CDF of  $x$  if a Random variable  $X$  has the PDF  $f(x)$  given by **07**

$$f(x) = \begin{cases} cx, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

**(b)** What is the role of Modular arithmetic in Computer security? State and prove Fermat's theorem and its application in Computer security **07**

**OR**

**Q.5 (a)** Assume a random variable  $X$  denotes the no of neurons in an experiment on a human brain. **07**  
If the random variable  $X$  takes the values 1, 2, 3 and 4 such that  $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$ , find the probability distribution function and cumulative distribution function of  $X$ .

**(b)** List and explain the metrics to evaluate Machine Learning Application algorithms **07**

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