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## GUJARAT TECHNOLOGICAL UNIVERSITY ME - SEMESTER-II(NEW)-EXAMINATION - WINTER-2020

## Subject Code:3720216

Date: 28/Jan/2021
Subject Name: Advance Algorithms
Time: 02:00 PM To 04:00 PM
Total Marks: 56

## Instructions:

1. Attempt any FOUR questions out of EIGHT questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
Q. 1 (a) 1. Write selection sort algorithm to sort the characters of string 'advance algorithms'.
4. Consider a network of n nodes. There is a cut C on the network. Prove that the flow $f(C)$ across any cut $C$ is equal to the total network flow value |f|.
(b) List properties of Matroid. Let $\mathrm{E}=\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$. Let $\mathrm{I}=\{\phi,\{\mathrm{A}\},\{\mathrm{B}\}$, $\{\mathrm{C}\},\{\mathrm{B}$, $\mathrm{C}\}\}$. Using the properties of matroid, check that $\mathrm{M}=(\mathrm{E}, \mathrm{I})$ is a matroid?
Q. 2 (a) What is triangular matrix. Prove that (1) the product of two lower-triangular matrices is lower-triangular, (2) the determinant of a lower-triangular or uppertriangular matrix is equal to the product of its diagonal elements and (3) the inverse of a lower-triangular matrix, if exists, is lower-triangular.
(b) Generate shortest path using BFS from source node 0 to destination node 5 .

Q. 3 (a) Find the maximum flow of the following network using Edmonds-Karp's maximum flow algorithm. Consider node 5 as source node and node 6 as sink node.

(b) 1. Define following terms: maximal matching, perfect matching, alternating $\mathbf{0 4}$
5. Generate maximal matchings from the following graph. $\mathbf{0 3}$

Q. 4 (a) 1. Explain various storage representations of directed and undirected graphs.
6. Discuss accounting method of amortized analysis with suitable example.
(b) Following are the two DNA sequences:
$\mathrm{X}=\mathrm{ACGCCGTA}$
$\mathrm{Y}=\mathrm{CGT}$
Find a maximum length common sub-sequence of X and Y using dynamic programming method.
Q. 5 (a) 1. Explain the key concepts used to prove that the given problem is NPcomplete problem.
7. Explain the different ways to represent polynomials.
(b) Mention the major steps for solving systems of congruences with the Chinese remainder theorem. Solve the following systems of congruences using Chinese remainder theorem.
$x=1(\bmod 3)$
$x=4(\bmod 5)$
$x=6(\bmod 7)$
Q. 6 (a) Write recursive FFT algorithm and explain how is it useful to compute DFT of an $n$-element vector $a=\left(a_{0} a_{1}, \ldots ., a_{n-1}\right)$, where $n$ is a power of 2 ?
(b) Find shortest path between all pairs of following graph using Floyd-Warshall algorithm.

Q. 7 (a) Solve the following linear program using Simplex method.

Maximize $18 \mathrm{x}_{1}+12.5 \mathrm{x}_{2}$
subject to

$$
\begin{aligned}
\mathrm{x}_{1}+\mathrm{x}_{2} & \leq 20 \\
\mathrm{x}_{1} & \leq 12 \\
& \leq 12 \\
\mathrm{x}_{2} & \leq 16 \\
\mathrm{x}_{1}, \mathrm{x}_{2} & \geq 0 .
\end{aligned}
$$

(b) Convert the following linear program into standard form.

Minimize $2 \mathrm{x}_{1}+7 \mathrm{x}_{2}$
subject to

$$
\begin{aligned}
\mathrm{x}_{1} & =7 \\
3 \mathrm{x}_{1}+\mathrm{x}_{2} & \geq 24 \\
\mathrm{x}_{2} & \geq 0
\end{aligned}
$$

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x_{3} \leq 0 .
$$

Q. 8 (a) Convert the following linear program into slack form.

Maximize 2x1-6x3
subject to

$$
\begin{aligned}
\mathrm{x}_{1}+\mathrm{x}_{2}-\mathrm{x}_{3} & \leq 7 \\
3 \mathrm{x}_{1}-\mathrm{x}_{2} & \geq 8 \\
-\mathrm{x}_{1}+2 \mathrm{x}_{2}+2 \mathrm{x}_{3} & \geq 0 \\
\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3} & \geq 0
\end{aligned}
$$

(b) Explain with example that how can you solve vertex-cover problem with $\mathbf{0 7}$ approximation algorithm.

