

GUJARAT TECHNOLOGICAL UNIVERSITY
ME - SEMESTER-1 (NEW) EXAMINATION – WINTER 2018

Subject Code: 3710812

Date: 01/01/2019

Subject Name: Computational Method for Mechanical Engineering

Time: 02:30 PM To 05:00 PM

Total Marks: 70

Instructions:

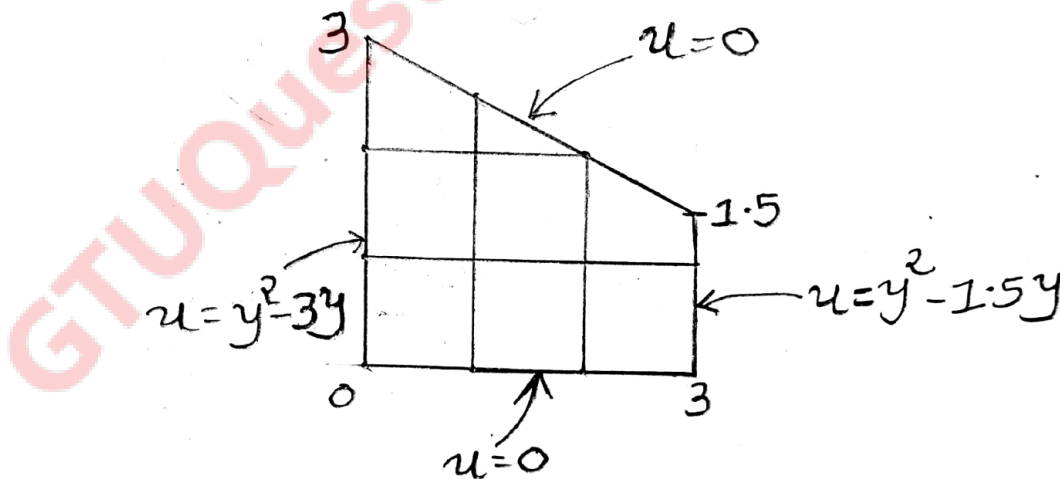
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full mark.

- Q.1 (a)** Consider the motion of a fluid flow through rectangular box B having no source and sink with in it. The dimensions of rectangular box are $\Delta x, \Delta y, \Delta z$. The velocity vector of fluid motion is $\vec{v} = v_1 i + v_2 j + v_3 k$. Using divergence of vector field, derive the condition for the conservation of mass, **07**

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{v}) = 0.$$

- (b)** Find the second order homogeneous linear differential equation for which the functions $x^2, x^2 \log x$ are the solution. **07**

- Q.2 (a)** Solve the Poisson equation $\nabla^2 u = 2$ for the figure shown below. Take a step size =1 along both directions. **07**



- (b)** For the differential equation $\frac{dy}{dx} = x^2 y + x^2$ and $y(1) = 1, y(1.1) = 1.2330, y(1.2) = 1.5485, y(1.3) = 1.9789$. Determine $y(1.4)$ using (1) Milne's Method and (2) Adams-Bashforth method. Compare the result obtained by both methods. Take accuracy up to four decimal places. **07**

OR

- (b) Obtain natural cubic spline for given data set, 07

x	2	3	4
y	11	49	123

Find the polynomial for interval [2, 3] and [3, 4]. Calculate $y(2.5)$ and $y'(2)$.

- Q.3 (a) If one solution of 2nd order homogeneous linear differential equation, $y'' + py' + qy = 0$ is y_1 . Using reduction of order, find the second solution y_2 for the differential equation. 07

- (b) An LTI system is described by the differential equation, $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 6x(t)$. Determine the forced and natural part of response of system when input $x(t) = u(t)$ with system initial conditions as $y(0) = 1, \frac{dy(t)}{dt}\bigg|_{t=0} = 2$. 07

OR

- Q.3 (a) The outflow of water from a cylindrical tank with a hole at the bottom. Find the height of water in the tank at any time if the tank has 2 cm diameter and hole has diameter 1 cm. The initial height of the water, when hole opened, is 2.25 m. When will the tank be empty? 07

- (b) Using Laplace transforms, Solve the differential equation $y'' + 2y' + 5y = 50t - 150, y(3) = -4, y'(3) = 14$. 07

- Q.4 (a) Develop the mathematical model for the membrane. 07

- (b) Represent the ordinary differential equation $y'' - 2y' + (1 + \lambda)y = 0, y(0) = 0, y(1) = 0$ in the form of Sturm-Liouville equation, if possible. Find the Eigen values and Eigen function for the differential equation. 07

OR

- Q.4 (a) Find the temperature distribution in rectangular plate having length "a" and width "b". The boundary conditions are: side at $x = 0, y = 0$ and $x = a$ is maintained at zero temperature and at $y = b, u = f(x)$. 07

- (b) Explain graphically the difference between even and odd function. Develop the Fourier series for the odd and even function. Find the Fourier series of function $f(x) = x + \pi, \text{ if } -\pi < x < \pi \text{ and } f(x + 2\pi) = f(x)$. 07

- Q.5 (a) What type of conic section the quadratic equation $7x_1^2 + 6x_1x_2 + 7x_2^2 = 200$ represents? Transform it to principal axis. 07

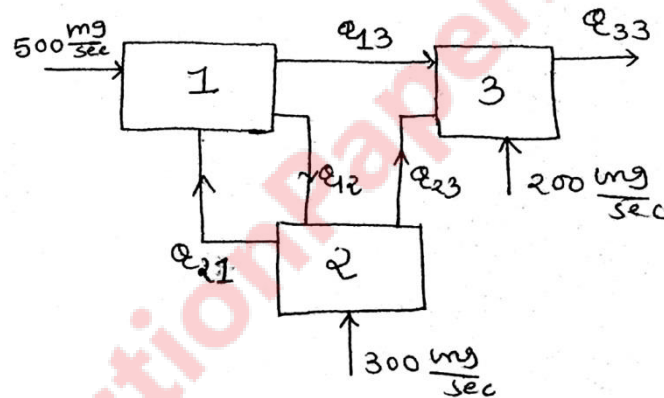
- (b) Find the regression line of y on x for the data point given in table as below, 04

Brinell Hardness, x	200	300	400	500
Tensile strength, (1000 psi), y	110	150	190	280

- (c) A quality inspector wants to draw random sample of two gaskets from a box containing 10 gaskets, three of which are defective. Find the probability function of random variable $x =$ number of defective in sample for the case sampling with replacement. 03

OR

- Q.5** (a) Three tanks are connected by pipes. The rate of mass transfer through each pipe is equal to the product of flow and concentration of the tank from which the flow originates. $Q_{33} = 120$, $Q_{13} = 40$, $Q_{12} = 90$, $Q_{23} = 60$ and $Q_{21} = 30$ units. Find the concentration of each tank. Use matrix inversion method. 07



- (b) Five independent measurements of the point of inflammation of diesel gave the values (in $^{\circ}F$), 144, 147, 146, 142, 144. Assume the normality, determine a 99% confidence interval of Mean. The value of t -distribution is 4.6. 04
- (c) Using a binomial distribution, compute the probability of obtaining at least two "six" in rolling a fair die 4 times. 03
