

**GUJARAT TECHNOLOGICAL UNIVERSITY**  
**ME – SEMESTER-II(NEW)-EXAMINATION – WINTER-2020**

**Subject Code: 3720801**

**Date: 03/Feb/2021**

**Subject Name: FINITE ELEMENT ANALYSIS**

**Time: 02:00 PM to 04:00 PM**

**Total Marks: 56**

**Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Machine design data book is allowed.

Q.1 (a) Derive element stiffness matrix for three node quadratic element as; 07

$$k^e = \frac{E_e A_e}{l_e} \begin{bmatrix} 7 & 1 & -8 \\ 1 & 7 & -8 \\ -8 & -8 & 16 \end{bmatrix}$$

Where,  $E_e$ ,  $A_e$  and  $l_e$  is Young's modulus, cross sectional area and length of element.

(b) For the truss shown in Figure 1, solve for the horizontal and vertical components of displacement at node 1. Also determine the stress in element 1. 07  
 Let  $A = 1 \text{ in}^2$ ,  $E = 10.0 \times 10^6 \text{ psi}$ , and  $L = 100 \text{ in}$ .

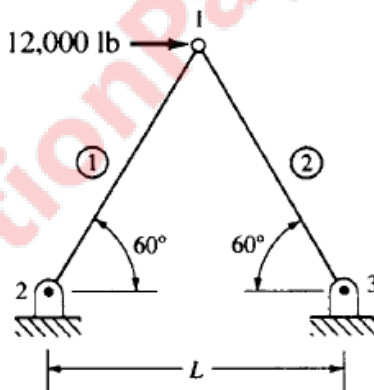


Figure 1

Q.2 (a) Discuss the types of elements commonly employed in practice for finite element analysis. 07

(b) For the system of spring shown in Figure 2, derive equilibrium equation of a system (in the form of  $KQ = F$ ) using total potential energy approach. Assume extension of the springs 1, 2, 3 and 4 as  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$  and  $\delta_4$  respectively. 07

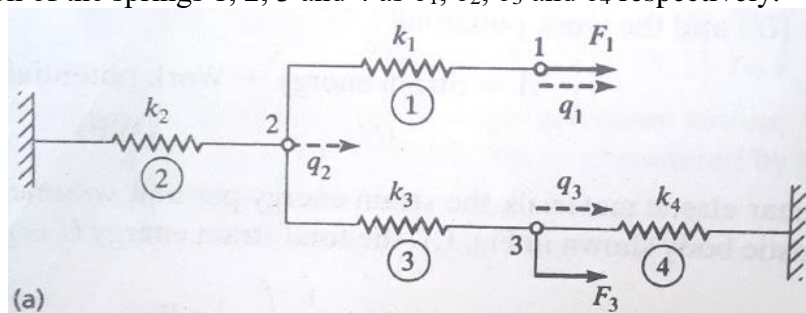


Figure 2

- Q.3 (a) An axial load  $P = 300 \times 10^3$  is applied at  $20^\circ\text{C}$  to the rod as shown in Figure 3. The temperature is then raised to  $60^\circ\text{C}$ . Determine global stiffness matrix, global load vector, nodal displacements and element stresses. 07

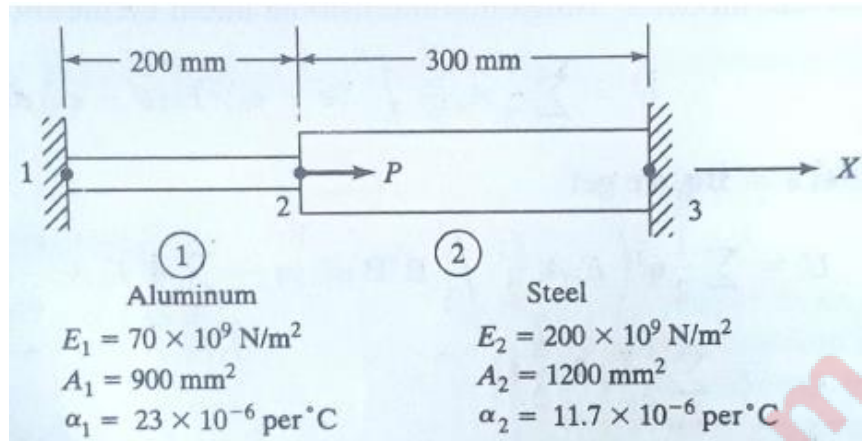


Figure 3

- (b) (i). Evaluate shape functions  $N_1$ ,  $N_2$  and  $N_3$  at the interior point P with coordinates (3.85, 4.8) for triangular element having coordinates of nodes 1, 2 and 3 as (1.5, 2); (7, 3.5) and (4, 7) respectively. 04  
(ii). Also determine the Jacobian of transformation  $\mathbf{J}$  and area for the triangular element. 03
- Q.4 (a) A composite wall consist of three materials, as shown in Figure 4. The outer temperature is  $T_0 = 20^\circ\text{C}$ . Convection heat transfer takes place on the inner surface of the wall with  $T_\infty = 800^\circ\text{C}$  and  $h = \text{W/m}^2^\circ\text{C}$ . Determine the temperature distribution in the wall. 04

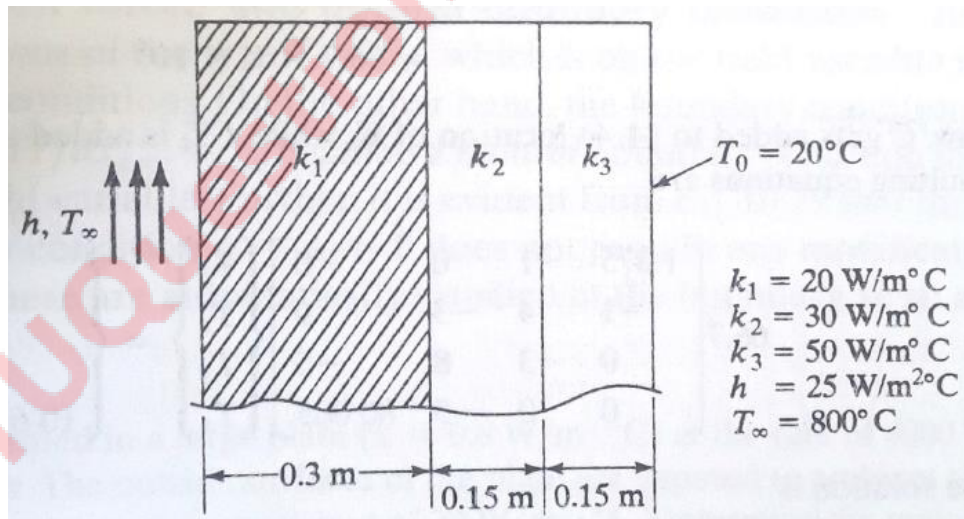


Figure 4

- (b) (i). Discuss shape function for four node rectangular element using. 04  
(ii). Define plane stress and plane strain problem. 03
- Q.5 (a) Derive mass matrix for 1D bar element and CST element. 07  
(b) Define various types of boundary conditions and discuss elimination approach for treatment of boundary conditions in finite element analysis. 07
- Q.6 (a) Enlist various methods for solution of Eigenvalue problem and discuss any one method in detail. 07  
(b) Derive governing partial differential equation for an isotropic, thin-plate 07

bending behavior as per Kirchhoff plate theory.

- Q.7 (a) For the fixed hinged beam subjected to a force and a moment as shown in Figure 5, determine governing equation in the form of  $F=KQ$  and show application of boundary condition. Assume  $EI$  to be constant throughout the beam. A force of 1000 lb and a moment of 1000 lb-ft are applied to the beam at midlength. The left end is a fixed support and the right end is a pin support. (F is global load vector, K is global stiffness matrix and Q is global displacement vector) 07

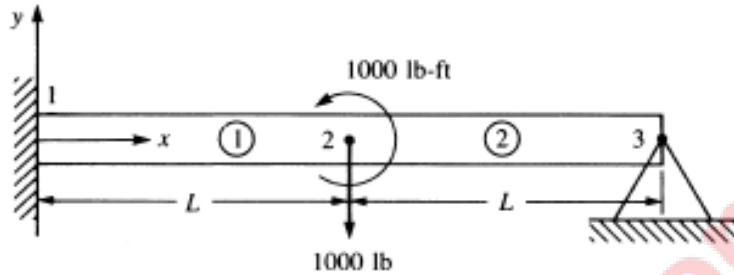


Figure 5

- (b) (i). Write steps for solution of structural problem using finite element analysis. 04  
(ii). Discuss shear locking phenomena with respect to CST element. 03
- Q.8 (a) Using direct stiffness method, determine stiffness matrix, displacement and slope at various nodes for propped cantilever beam shown in Figure 6. Propped cantilever beam is subjected to end load P. Beam is assumed to have constant  $EI$  and length  $2L$ . It is supported by a roller at mid length and is built in at the right end. 07

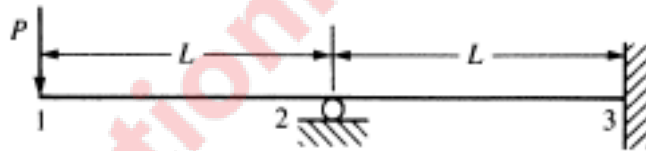


Figure 6

- (b) (i). Enlist the various sources of non-linearity. 02  
(ii). Discuss material non-linearity in detail. 05

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