Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–III (NEW) EXAMINATION – WINTER 2020

Subject Code:3130006 **Date:09/03/2021**

Subject Name:Probability and Statistics

Time:10:30 AM TO 12:30 PM **Total Marks:56**

Instructions:

1. **Attempt any FOUR questions out of EIGHT questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Statistical Tables are required.**

Q.1 **(a)** Find the mean, median and Mode for the following frequency distribution: **03**

x	1	2	3	4	5	6	7	8	9	10
f	4	7	8	10	6	6	4	2	2	1

(b) An insurance company insured 2000 bike drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a bike driver, a car driver and a truck driver is 0.10, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a bike driver? **04**

(c) (i) A manufacturer of external hard drives claims that only 10 % of his drives require repairs within the warranty period of 12 months. If 5 of 20 of his drives required repairs within the first year, does this tend to support or refute the claim? **03**

(ii) The actual amount of instant coffee that a filling machine puts into "4 −ounce" jars may be looked upon as a random variable having a normal distribution with $\sigma = 0.04$ ounce. If only 2 % of the jars are to contain less than 4 ounces, what should be the mean fill of these jars? Out of 10000 jars sold, how many are expected to contain more than 4.2 ounces? **04**

Q.2 **(a)** If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Find the probability that out of 2000 individuals, (i) more than 2 individuals; (ii) exactly 3 individuals will suffer a bad reaction. **03**

(b) A stenographer claims that she can type at the rate of 120 words per minute. She demonstrated, on the basis of 100 trials, an average speed of 116 words with a standard deviation of 15 words. Does this enable us to reject the null hypothesis $\mu = 120$ against the alternative hypothesis $\mu < 120$ at the 0.05 level of significance? **04**

(c) (i) The time to check out and process payment information at an office supplies Web site can be modeled as a random variable with mean $\mu = 63$ seconds and variance $\sigma^2 = 81$ seconds. What is the probability that a random sample of size 36 has mean greater than 66.75? **03**

(ii) If two random variables X and Y have the joint density **04**

$$f(x,y) = \begin{cases} k(x+y^2), & for\ 0 < x < 1, 0 < y < 1 \\ 0, & elsewhere \end{cases},$$

find k and the mean of the conditional density $f_1(x \mid 0.5)$ where $f_1(x)$ is the marginal probability density of X.

Q.3 **(a)** If A and B are independent events with $P(A) = 0.26$, and $P(B) = 0.45$, find $(a)\ P(A \cap B)$; $(\ b\ )\ P(A \cap \bar{B})$; $(\ c\ )\ P(\bar{A} \cap \bar{B})$. **03**

(b) Compute Karl Pearson's coefficient of correlation between X and Y for the following data: **04**

X	100	98	78	85	110	93	80
Y	85	90	70	72	95	81	74

(c) (i) The arithmetic means of runs scored by three batsmen A, B and C, in the same series of 10 innings, are 50, 48 and 12 respectively. The standard deviations of their runs are 15, 12 and 2 respectively. Who is the most consistent of the three? **03**

(ii) Calculate the first four moments about the mean of the following data: **04**

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

Q.4 **(a)** The following table gives the probabilities that a certain computer will malfunction 0, 1, 2, 3, 4, 5 or 6 times on any one day: **03**

Number of malfunctions	x :	0	1	2	3	4	5	6
Probability	$f(x)$:	0.17	0.29	0.27	0.16	0.07	0.03	0.01

Find the mean and variance of this probability distribution.

(b) The coefficient of rank correlation of marks obtained by 10 students in English and Economics was found to be 0.6. It was later discovered that the difference in ranks in the two subjects obtained by one of the students was wrongly taken as 7 instead of 1. Find the correct coefficient of rank correlation. **04**

(c) (i) Find out mean deviation about median for the following series: **03**

Size	4	6	8	10	12	14	16
Freq.	2	1	3	6	4	3	1

(ii) Find Karl Pearson's coefficient of skewness for the following data: **04**

x	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$
f	13	20	30	25	12

Q.5 **(a)** The life in hours of a certain kind of radio tube has the probability density **03**

$$f(x) = \begin{cases} 100/x^2, & for\ x \geq 100 \\ 0, & elsewhere \end{cases},$$

find the distribution function and use it to determine the probability that the life of tube is more than 150 hrs.

(b) The number of flaws in a fiber optic cable follows a Poisson process with an average of 0.6 per 100 feet. **04**

(i) Find the probability of exactly 2 flaws in a 200 foot cable.

(ii) Find the probability of exactly 1 flaw in the first 100 feet and exactly 1 flaw in the second 100 feet.

(c) The population (p) of a small community on the outskirts of a city grows rapidly over a $20-$year period: **07**

t	0	5	10	15	20
p	100	200	450	950	2000

As an engineer working for a utility company, you must forecast the population 5 years into the future in order to anticipate the demand for power. Employ an exponential model and linear regression to make this prediction.

Q.6 **(a)** The joint probability density of two random variables is given by **03**

$$f(x_1, x_2) = \begin{cases} 6e^{-2x_1 - 3x_2}, & for\ x_1 > 0, x_2 > 0 \\ 0, & elsewhere \end{cases}.$$

Find the marginal densities of both the random variables and hence show that the two random variables are independent.

(b) The probability that an electronic component will fail in less than 1000 hours of continuous use is 0.25. Use the normal approximation to find the probability that among 200 such components fewer than 45 will fail in less than 1000 hours of continuous use. **04**

(c) Fit a parabola $y = a + bx + cx^2$ to the following data: **07**

x	1	2	3	5	6
y	1.1	5.8	17.5	55.9	86.7

Q.7 **(a)** In a study of automobile collision insurance costs, a random sample of 80 body repair costs for a particular kind of damage had a mean of $33065\ Rs.$ and a standard deviation of $4364\ Rs.$ If $\bar{x} = 33065\ Rs.$ is used as a point estimate of the true average repair cost of this kind of damage, with what confidence can one assert that the error does not exceed $700\ Rs.$? **03**

(b) In a certain city, the daily consumption of electric power (in millions of kilowatt-hours) can be treated as a random variable having a gamma distribution with $\alpha = 2$ and $\beta = 3$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day? Also, find the mean of this probability density. **04**

(c) (i) Ten bearings made by a certain process have a mean diameter of $0.506\ cm$ and a standard deviation of $0.004\ cm$. Assuming that the data may be looked upon as a random variable from a normal population, construct a 95 % confidence interval for the actual average diameter of bearings made by this process. **03**

(ii) A consumer protection agency wants to test a paint manufacturer's claim that the average drying time of his new paint is 20 minutes. It instructs a member of its research staff to paint each of 36 boards using a different $1-$gallon can of the paint, with the intention of rejecting the claim if the mean of the drying times exceeds 20.75 minutes. Otherwise, it will accept the claim. Find the probability of a Type I error. Also, find the probability of a Type II error when $\mu = 21$ minutes. Assume that $\sigma = 2.4$ minutes. **04**

Q.8 **(a)** The dean of a college wants to use the mean of a random sample to estimate the average amount of time students take to get from one class to the next, and she wants to be able to assert with 99 % confidence that the error is at most 0.25 minute. If it can be presumed from experience that $\sigma = 1.40$ minutes, how large a sample will she have to take? **03**

(b) How exponential distribution is useful in real applications? Find the mean and variance of the exponential distribution **04**

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & for\ x > 0, \beta > 0 \\ 0, & elsewhere \end{cases}.$$

(c) A random sample from a company's very extensive files shows that orders for a certain piece of machinery were filled, respectively, in $10, 12, 19, 14, 15, 18, 11$ and 13 days. Use the level of significance $\alpha = 0.01$ to test the claim that on average such orders are filled in 10.5 days. Choose the alternative hypothesis so that rejection of the null hypothesis $\mu = 10.5$ implies that it takes longer than indicated. Assume normality. **07**

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