Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE- SEMESTER–IV (NEW) EXAMINATION – WINTER 2020**

**Subject Code:3140708 Date:17/02/2021**

**Subject Name:Discrete Mathematics**

**Time:02:30 PM TO 04:30 PM Total Marks:56**

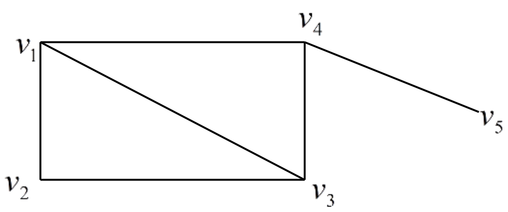
**Instructions:**

1. **Attempt any FOUR questions out of EIGHT questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

| | | | Marks |
|---|---|---|---|
| **Q.1** | **(a)** | Find the power sets of $(i)\{a\}$ , $(ii)\{a,b,c\}$. | 03 |
| | **(b)** | If $f(x)=2x, g(x)=x^2, h(x)=x+1$ then find *(fog)oh* and *fo(goh)*. | 04 |
| | **(c)** | (i) Let N be the set of natural numbers. Let R be a relation in N defined by $x$R$y$ if and only if $x+3y=12$. Examine the relation for (i) reflexive (ii) symmetric (iii) transitive. | 03 |
| | | (ii) Draw the Hasse diagram representing the partial ordering *{(a,b)/ a* divides *b*} on {1,2,3,4,6,8,12}. | 04 |
| **Q.2** | **(a)** | Let R be a relation defined in A={1,2,3,5,7,9} as R={(1,1), (1,3), (1,5), (1,7), (2,2), (3,1), (3,3), (3,5), (3,7), (5,1), (5,3), (5,5), (5,7), (7,1), (7,3), (7,5), (7,7), (9,9)}. Find the partitions of A based on the equivalence relation R. | 03 |
| | **(b)** | In a box there are 5 black pens, 3 white pens and 4 red pens. In how many ways can 2 black pens, 2 white pens and 2 red pens can be chosen? | 04 |
| | **(c)** | Solve the recurrence relation $a_n - 4a_{n-1} + 4a_{n-2} = n + 3^n$ using undetermined coefficient method. | 07 |
| **Q.3** | **(a)** | Define self-loop, adjacent vertices and a pendant vertax. | 03 |
| | **(b)** | Define tree. Prove that if a graph *G* has one and only one path between every pair of vertices then *G* is a tree. | 04 |
| | **(c)** | (i) Find the number of edges in G if it has 5 vertices each of degree 2. | 03 |
| | | (ii) Define complement of a subgraph by drawing the graphs. | 04 |
| **Q.4** | **(a)** | Show that the algebraic structure *(G,*)* is a group, where $G=\{(a,b)/a,b\in R, a\neq 0\}$ and * is a binary operation defined by $(a,b)*(c,d)=(ac,bc+d)$ for all $(a,b),(c,d)\in G$. | 03 |
| | **(b)** | Define path and circuit of a graph by drawing the graphs. | 04 |
| | **(c)** | (i) Show that the operation * defined by $x*y=x^y$ on the set N of natural numbers is neither commutative nor associative. | 03 |
| | | (ii) Define ring. Show that the algebraic system $(Z_9,+_9,\bullet_9)$, where $Z_9=\{0,1,2,3,\ldots,8\}$ under the operations of addition and multiplication of congruence modulo 9, form a ring. | 04 |

**Q.5** **(a)** Define subgraph. Let $H$ be a subgroup of $(Z,+)$, where H is the set of even integers and $Z$ is the set of all integers and + is the operation of addition. Find all right cosets of $H$ in $Z$. **03**

**(b)** Define adjacency matrix and find the same for **04**




**(c)** (i) Draw the composite table for the operation * defined by $x*y=x$, $\forall x, y \in S = \{a, b, c, d\}$. **03**

(ii) Show that an algebraic structure $(G,\bullet)$ is an abelian group, where **04**

$G=\{A,B,C,D\}$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$

$D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\bullet$ is the binary operation of matrix multiplication.

**Q.6** **(a)** Define indegree and outdegree of a graph with example. **03**

**(b)** Prove that the inverse of an element is unique in a group $(G,*)$. **04**

**(c)** (i) Does a 3-regular graph with 5 vertices exist? **03**

(ii) Define centre of a graph and radius of a tree. **04**

**Q.7** **(a)** Check the properties of commutative and associative for the operation * defined by $x*y=x+y-2$ on the set $Z$ of integers. **03**

**(b)** Define group permutation. Find the inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$. **04**

**(c)** (i) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. **03**

(ii) Obtain the d.n.f. of the form $(p \rightarrow q) \wedge (\neg p \wedge q)$. **04**

**Q.8** **(a)** Find the domain of the function $f(x) = \sqrt{16 - x^2}$. **03**

**(b)** Define lattice. Determine whether POSET $\{\{1,2,3,4,5\};|\}$ is a lattice. **04**

**(c)** Show that the propositions $\neg(p \wedge q)$ and $\neg p \vee q$ are logically equivalent. **07**

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